A coupled transilience model for turbulent air flow within plant canopies and the planetary boundary layer

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Abstract

Stull's transilient turbulence theory (T-theory) was originally developed to study turbulent air flow in the planetary boundary layer (PBL). The present work extends this model to study the interaction of turbulent air flow within a plant canopy and in the PBL. Additional mechanisms for the causes of wind shear-like leaf drag and destabilizing air flow, such as solar radiation absorption by leaves, which in turn warms the surrounding air, are included. Stull's key concept of transilient coefficients, which describe nonlocal eddy transport, is retained. However, the precise calculations of these coefficients have been modified according to the physical situation.

The numerical simulations based on our coupled model are capable not only of simulating turbulent air flow in the PBL, but they can also reproduce the counter-thermal gradient flows within plant canopies and agree well with experimental observation data collected in the Black Moshannon Forest in Pennsylvania, USA. © 1997 Published by Elsevier Science B.V.

Keywords: Turbulent airflow; Plant canopies; Planetary boundary layer; Transilient coefficients

1. Introduction

Most land surfaces are covered by vegetation. The leaves of the plant canopy absorb solar radiation, which warms the surrounding air. Moreover, when air flows near the vegetation, there is frictional resistance. Both of these factors cause additional turbulence in atmospheric air flow around the plant canopy.

Typically, such plant canopy effects are mostly felt from the ground up to $2h_c$, where $h_c$ is the canopy height. This is known as the roughness sublayer. For a very sparse canopy, it may even reach $5h_c$ (Raupach et al., 1991). Thus, over time, we expect that the mean horizontal wind speed, $U$, will vary rapidly with height from the ground in the roughness sublayer (especially within the plant canopy due to the effects of leaf drag), and that the changes will become more moderate outside it. In fact, experimental measurements have confirmed that $U$ is strongly sheared at the canopy height and attenuates quickly within the canopy.

This strongly sheared velocity profile favors turbulence and eddy formation. Experimental studies on agricultural crops (Wilson et al., 1982, Shaw et al., 1974), wind tunnels (Raupach, 1989), and forest canopies (Raupach et al., 1989, Denmead and Bradley, 1987) have shown that turbulence within and just above plant canopies is dominated by highly coherent eddies with length scales that are at least as large as the canopy height.
height. They account for most of the vertical transport of momentum and scalar properties such as heat, water vapor and carbon dioxide concentrations within the canopy.

It is therefore natural to consider that the plant canopy is immersed in the atmospheric boundary layer, which has even larger eddies that scale to the boundary layer depth. By studying their mutual interaction, a more accurate model can be developed. This is the purpose of this paper. We first review some related models.

For a long time the conventional gradient-diffusion theory (K-theory) has been used to study the turbulent air flow within and above plant canopy. Calculations based on the K-theory, however, do not agree well with experimental data (Legg and Monteith, 1975; Denmead and Bradley, 1985, 1987). The theory dictates that heat flux goes locally down the temperature gradient. When vegetation is present, the temperature reaches a peak in the upper part of the canopy due to solar radiation absorption by the leaves. According to the K-theory, this gives rise to a downward heat flux below the level of peak temperature. However, experimental measurements show that the heat flux is often upward throughout the whole canopy. This counter-thermal gradient flux cannot be simulated by the classical K-theory.

To remedy this shortcoming, Miller et al. (1991) and Li et al. (1985, 1990)) attempted to include the effects of large-scale eddies in a first-order closure model. They modified the gradient diffusion parametrization of the Reynolds stress by including an additional bulk momentum transport term for larger-scale turbulence. Higher-order closure techniques (Wilson and Shaw, 1977; Shaw et al., 1983; Meyer and Paw U, 1986, 1987) and Lagrangian models (Raupach, 1987, 1989) have also been proposed as improvements. These models have overcome some of the drawbacks of the K-theory, but in the case of counter-thermal gradient flows, quantitative predictions are still far from satisfactory. This may be due to the fact that eddy transport in the atmosphere is a convection phenomenon, rather than a diffusion mechanism.

A nonlocal closure method for modeling turbulent air flow in the planetary boundary layer (PBL), called the transilient turbulence theory (T-theory), has been developed by Stull and co-workers (Stull, 1984, 1986, 1994a,b; Stull and Hasagawa, 1984; Stull and Driedonks, 1987). In this theory, it is assumed that finite-sized eddies, as opposed to diffusion, transport fluid properties like temperature and momentum. Large eddies are responsible for long-distance transport, while small eddies are responsible for short-distance transport. That the model is nonlocal comes from this long-distance interaction.

In the original T-theory model, the plant canopy structure was treated as a horizontally uniform bottom boundary. The influence of the plant canopy would therefore be imposed through boundary conditions only. The role of the complex canopy structure on the turbulent air flow around it was oversimplified.

In this paper we extend this theory to include the detailed mechanisms that affect turbulent air flow within and above the plant canopy. These are then coupled with the flow in the PBL. The coupled T-theory model gives better predictions of transport both within the canopy and in the PBL.

2. The coupled T-theory model

Since we need to couple the air flow near the plant canopy with that in the PBL above it, we first describe the governing equations in the two regions.

2.1. Governing equations in the PBL

For simplicity, we ignore the effects of radiative flux divergence and latent heat release associated with the moisture in the PBL. The principal mechanisms for turbulence generation in the PBL are wind shear and buoyancy. The medium and smaller-scale turbulence generated by solar heating of the vegetation and the frictional effects of the plant canopy structure are restricted to the air mass near the plant canopy, and are described in Section 2.2. However, the resulting turbulent kinetic energy can be carried to the PBL from the plant canopy via large eddies. The turbulence is then dissipated by the small eddies because of air viscosity.
Let \( x \) and \( y \) represent the horizontal, and \( z \) the vertical directions. \( U, V \) are the \( x, y \) components of the mean (over time) wind velocity vector, respectively. \( U_g, V_g \) are the \( x, y \) components of the geostrophic wind velocity vector, respectively. \( u', v' \) and \( w' \) are the corresponding departures from the \( x, y \) and \( z \) components of the mean wind velocity, respectively. \( \Theta \) is the potential temperature, \( Q \) is the mean water vapor specific humidity (g kg\(^{-1}\)). \( \theta' \) is the departure of the potential temperature from its mean value, and finally \( q' \) is the departure of the specific humidity from its mean value.

\( \overline{u'w'} \) and \( \overline{v'w'} \) are the \( x, y \) components of the vertical turbulent momentum flux, respectively, where the overlines represent the corresponding means over time. Similarly, \( \overline{\theta'w'} \) is the sensible heat flux, and \( \overline{q'w'} \) is the latent heat flux in the vertical direction. Finally, we let \( f = 1.45 \times 10^{-4} \sin(\phi) \), where \( \phi \) is the latitude of the location under consideration. This is the Coriolis effect on the PBL.

The one-dimensional governing equation for \( U \) can be cast as (Stull, 1991)

\[
\frac{\partial U}{\partial t} = f(V - V_g) - \frac{\partial \overline{u'w'}}{\partial z},
\]

which is discretized by dividing the vertical column of air with unit cross-section area into \( n \) grid cells, which can be unevenly sized. Let \( \Delta t \) be the time step and \( \Delta z \) the cell thickness located at height \( z \) above the ground. Standard discretization procedures yield

\[
U(t + \Delta t, z) = U(t, z) + \Delta t f(V(t, z) - V_g(t, z)) - \Delta t \overline{u'w'}(t, z + \Delta z) - \overline{u'w'}(t, z).
\]

In order to close the model, the turbulent momentum flux \( \overline{u'w'} \) has to be related to some known quantities. In Stull's transilient turbulence theory, this is resolved by introducing new variables known as transilient coefficients, \( C_{ij}(t, \Delta t) \). \( C_{ij} \) is defined as the portion of air in the destination cell \( i \) at time \( t + \Delta t \) that comes from the source cell \( j \) at time \( t \). It is instructive to look at an example.

**Example:** Let the masses of air in the cells 2 and 4 be 3 and 5 kg, respectively. The masses in the cells are constant in time. If \( 1/6 \) of the air in cell 2 at time \( t \) moves to cell 4 at time \( t + \Delta t \), then \( C_{42}(t, \Delta t) = 3 \times 1/5 = 1/10 \).

It is clear from this definition that \( C_{ij} \geq 0 \). Moreover, we have (Stull, 1994a)

\[
\sum_i \frac{m_i}{m_j} C_{ij} = 1,
\]

\[
\sum_j C_{ij} = 1,
\]

where \( m_i \) is the mass of cell \( i \). This is a consequence of conservation.

We now return to the question of model closure. First, we number the grid cells 1 to \( n \) starting from the ground. Note that these governing equations in the PBL apply only to the cells outside the plant canopy; modified versions for within the plant canopy are described later. Let \( \Delta z_i \) be the thickness of the cell \( i \), which is proportional to \( m_i \). Let \( F_k(t) = \overline{u'w'} \) be the turbulent momentum flux across the top of the cell \( k \). Then for \( k + 1 \leq i \leq n \), cell \( i \) has an incoming (upward) flux,

\[
\frac{1}{\Delta t} \sum_{j=1}^{k} C_{ij} U_j \Delta z_i,
\]

from cells 1 to \( k \) crossing the top of the surface of cell \( k \). On the other hand, the outgoing (downward) flux through the top surface of cell \( k \) to cell \( i \) for \( 1 \leq i \leq k \) is

\[
\frac{1}{\Delta t} \sum_{j=k+1}^{n} C_{ij} U_j \Delta z_i.
\]
Hence the total net flux through the top surface of cell $k$ is

$$F_k(t, \Delta t) = \frac{1}{\Delta t} \left[ \sum_{i=k+1}^{n} \sum_{j=1}^{k} C_{ij}U_j\Delta z_i - \sum_{i=1}^{k} \sum_{j=k+1}^{n} C_{ij}U_j\Delta z_i \right] \text{ for } 1 \leq k \leq n. \tag{5}$$

Exchanging the dummy indices $i$ and $j$ in the first summation term, we have

$$F_k(t, \Delta t) = \frac{1}{\Delta t} \sum_{i=1}^{k} \sum_{j=k+1}^{n} (C_{ji}U_i\Delta z_j - C_{ij}U_j\Delta z_i) \text{ for } 1 \leq k \leq n. \tag{6}$$

Now, since

$$\sum_{i=1}^{n} \sum_{j=1}^{k} (C_{j}U_i\Delta z_j - C_{ij}U_j\Delta z_i) = 0,$$ by a simple exchange of the dummy index in the first term, we can use this result to rewrite Eq. (6) as

$$F_k(t, \Delta t) = \frac{1}{\Delta t} \sum_{i=1}^{k} \sum_{j=k+1}^{n} (C_{ji}U_i\Delta z_j - C_{ij}U_j\Delta z_i) \text{ for } 1 \leq k \leq n,$$ which immediately gives

$$F_k(t, \Delta t) - F_{k-1}(t, \Delta t) = \frac{1}{\Delta t} \sum_{j=1}^{n} \left[ C_{jk}U_k\Delta z_j - C_{kj}U_j\Delta z_k \right] \text{ for } 2 \leq k \leq n. \tag{7}$$

By means of Eqs. (3) and (4), since $m_i$ is proportional to $\Delta z_i$, the first term on the right-hand side of Eq. (8) can be written as

$$\sum_{j=1}^{n} C_{jk}U_k\Delta z_j = U_k \sum_{j=1}^{n} C_{jk}\Delta z_j = U_k \Delta z_k = \Delta z_k \sum_{j=1}^{n} C_{kj}U_k,$$ so Eq. (8) can be cast as

$$F_k(t, \Delta t) - F_{k-1}(t, \Delta t) = \frac{\Delta z_k}{\Delta t} \sum_{j=1}^{n} C_{kj}(t, \Delta t)(U_k(t) - U_j(t)) \text{ for } 2 \leq k \leq n. \tag{10}$$

We have to treat grid cell 1 somewhat differently because Eq. (6) does not hold for $k = 0$. The turbulent momentum flux $F_0$ at the ground should be known from the given boundary conditions.

Setting $k = 1$ in Eq. (7), we have

$$F_1(t, \Delta t) - F_0(t, \Delta t) = \frac{1}{\Delta t} \sum_{j=1}^{n} \left[ C_{j1}U_1\Delta z_j - C_{1j}U_j\Delta z_1 \right] = \frac{\Delta z_1}{\Delta t} \sum_{j=1}^{n} C_{1j}(U_1 - U_j) - F_0. \tag{11}$$

where we have again used Eqs. (3 and 4). Combining Eqs. (2) and (10), we have

$$U_i(t + \Delta t) = U_i(t) + \Delta tf \left[ V_i(t) - V_{z_i}(t) \right] - \sum_{j=1}^{n} C_{ij}(t, \Delta t)(U_i(t) - U_j(t))$$

$$= \Delta tf \left[ V_i(t) - V_{z_i}(t) \right] + \sum_{j=1}^{n} C_{ij}(t, \Delta t)U_j(t) \text{ for } 2 \leq i \leq n. \tag{12}$$

Here the dependence of $U$ on $z$ is represented by the subscript $i$, which denotes the cell number. The same remark applies for other equations.
Using Eqs. (2) and (11), we have
\[ U_i(t + \Delta t) = U_i(t) + \Delta t f[V_i(t) - V_g(t)] - \sum_{j=1}^n C_{ij}(t, \Delta t)[U_j(t) - U_i(t)] + F_0 \frac{\Delta t}{\Delta z_1} \]
\[ = \Delta t f[V_i(t) - V_g(t)] + \sum_{j=1}^n C_{ij}(t, \Delta t)U_j(t) + F_0 \frac{\Delta t}{\Delta z_1} \] for \( i = 1 \). (13)

Similar arguments can now be applied to other governing equations besides Eq. (1). To simplify our representation, we introduce the function
\[ \delta_{ii} = \begin{cases} 1, & \text{if } i = 1 \\ 0, & \text{if } i \neq 1 \end{cases} \] (14)

The governing equations are then
\[ U_i(t + \Delta t) = +f[V_i(t) - V_g(t)]\Delta t + \sum_{j=1}^n C_{ij}(t, \Delta t)U_j(t) + \delta_{ii}F_{0u} \frac{\Delta t}{\Delta z_1}, \] (15)
\[ V_i(t + \Delta t) = -f[U_i(t) - U_g(t)]\Delta t + \sum_{j=1}^n C_{ij}(t, \Delta t)V_j(t) + \delta_{ii}F_{0v} \frac{\Delta t}{\Delta z_1}, \] (16)
\[ \Theta_i(t + \Delta t) = \sum_{j=1}^n C_{ij}(t, \Delta t)\Theta_j(t) + \delta_{ii}F_{0\theta} \frac{\Delta t}{\Delta z_1}, \] (17)
\[ Q_i(t + \Delta t) = \sum_{j=1}^n C_{ij}(t, \Delta t)Q_j(t) + \delta_{ii}F_{0q} \frac{\Delta t}{\Delta z_1}, \] (18)

where \( F_{0u} \) and \( F_{0v} \) are the vertical momentum fluxes in the \( x \) and \( y \) directions on the ground, respectively, and \( F_{0\theta} \) and \( F_{0q} \) are the sensible heat and latent heat fluxes on the ground, respectively. We will discuss methods for obtaining \( C_{ij} \) in Section 3.

2.2. Governing equations within the plant canopy

Within the plant canopy, turbulence is generated from several sources. First, there is solar radiation absorption by the leaves and the ground surface, which in turn warms the surrounding air. The induced buoyancy of the warm air leads to the generation of turbulence. Another important factor is the frictional resistance to air flow due to the ground surface and the vegetation, which reduces the wind speed. This strongly sheared wind speed pattern favors eddy formation.

The complex patterns of leaves, branches, and individual trees cause the air flow in this roughness sublayer (which lies at the bottom of the PBL) to be strongly three-dimensional. To simplify our task, we assume a horizontally homogeneous air flow. Then, following the horizontal averaging procedures of Wilson and Shaw (1977) and Raupach and Shaw (1982), the three-dimensional turbulent flow can be simplified to equations in a one-dimensional framework as follows. Note that in the following, \( U \) is in fact a horizontally averaged value, usually denoted \( \langle U \rangle \) by other authors, as are all the variables appearing in the equations:
\[ \frac{dU}{dt} = f(V - V_g) - \frac{\partial \overline{uv}}{\partial z} - 0.5C_{ai} a[U^2 + V^2]^{1/2} U, \] (19)
\[ \frac{dV}{dt} = -f(U - U_g) - \frac{\partial \overline{uv}}{\partial z} - 0.5C_{ai} a[U^2 + V^2]^{1/2} V, \] (20)

\[ \frac{dW}{dt} = 0, \] (21)
\[
\frac{d\Theta}{dt} = \frac{\delta \Theta'}{\delta z} - \frac{\delta F_h}{\delta z},
\]
\[
\frac{dQ}{dt} = -\frac{\delta Q'}{\delta z} - \frac{\delta F_q}{\delta z},
\]
(21) (22)

where \( a \) is the foliage area volume density, \( C_{dl} = 0.2 \) (Kaimal and Finnigan, 1994) is the local drag coefficient of the leaves including the shelter factor. Hence the last terms on the right-hand sides of Eqs. (19) and (20) model the leaf drag. \( F_h \) and \( F_q \) are the averaged sensible heat and latent heat fluxes, respectively, across the leaf surfaces. All other terms are the same as in the equations for the PBL.

We now follow similar discretization procedures as for the governing equations for the PBL. Let \( L_a(z) \) be the downward accumulated leaf area index function of height \( z \), \( r_{ac}(z) \) the solar radiation attenuation function within the plant canopy, and \( r_a(z) \) the solar radiation absorbed by leaves per unit volume. Based on Beer’s law, the attenuation of solar radiation can be expressed as a function of \( L_a(z) \),

\[
r_{ac}(z) = r_a e^{-kL_a(z)},
\]
(23)

where \( k \) is the extinction coefficient. The radiation absorbed by leaves per unit volume can be approximated as

\[
r_a(z) = \frac{dr_{ac}}{dz} = kr_a(z) \left[ -\frac{dL_a(z)}{dz} \right],
\]
(24)

where \([-dL_a(z)/dz]\) is the foliage area volume density. Assuming that the solar radiation absorbed by each leaf is partitioned between sensible and latent heat fluxes (Monteith and Unsworth, 1990), the proportions of the sensible and latent heat fluxes can be obtained by the Bowen ratio method, which leads to

\[
-\frac{\delta F_h}{\delta z} = \frac{1}{\rho C_p} \frac{\beta}{1 + \beta} r_{ac}(z) = \frac{1}{\rho C_p} \frac{\beta}{1 + \beta} kr_{ac}(z) \left[ -\frac{dL_a(z)}{dz} \right],
\]
(25)

\[
-\frac{\delta F_q}{\delta z} = \frac{1}{\rho C_p} \frac{1}{1 + \beta} r_{ac}(z) = \frac{1}{\rho L_v} \frac{1}{1 + \beta} kr_{ac}(z) \left[ -\frac{dL_a(z)}{dz} \right],
\]
(26)

where \( \rho \) is the air density, \( \beta \) is the Bowen ratio (ratio of sensible to latent heat), and \( C_p \) is the specific heat of air, \( L_v \) is the latent heat of vapourization (vapour:liquid) at 20°C.

We can now assemble all our information to obtain the discretized form of the governing equations for turbulent air flow within the plant canopy. The equations are the same as Eqs. (15)–(18) except for some extra source terms. We can in fact view Eqs. (15)–(18) as special cases of Eqs. (28)–(31) (see below) if we take the precaution of switching off the extra source terms whenever the grid cell \( i \) does not lie within the plant canopy.

In view of the above remark, we let

\[
pc(i) = \begin{cases} 1, & \text{if grid cell } i \text{ is within the plant canopy} \\ 0, & \text{if grid cell } i \text{ is outside the plant canopy} \end{cases}
\]
(27)

Then the overall governing equations are

\[
U_i(t + \Delta t) = \sum_{j=1}^{n} C_{ij}(t, \Delta t) U_j(t) - pc(i) 0.5 C_{dl} a_i |U_i^2 + V_i^2|^{1/2} U_i \Delta t + \delta_{1i} F_0 u \frac{\Delta t}{\Delta z_i},
\]
(28)
3. Parametrization scheme of the transient coefficient matrix

We number the grid cells of an air column 1 to n starting from the ground, and employ Eqs. (28)–(31) in all the cells in view of the fact that we have defined the function \( p_c \) in Eq. (27). Such a coupling between the plant canopy and the PBL constitutes the basis for our coupled T-model for plant canopy flow. Since changes are more rapid near the plant canopy, the grid cells are usually unevenly sized, with a higher density near the ground and the canopy.

To implement the scheme, the transient coefficients \( C_{ij} \) for all \( 1 \leq i, j \leq n \) have to be parametrized. \( C_{ij} \) measures the mixing between cells \( i \) and \( j \) due to eddy movements. One can in fact obtain an estimate of the mixing using the turbulent kinetic energy equation (Stull and Driedonks, 1987). A quantity \( \chi_{ij} \), defined as the mixing potential between cells \( i \) and \( j \), can be computed using known quantities. Then \( C_{ij} \) is easily related to \( \chi_{ij} \).

Here we have followed Stull’s TKE parametrization approach to estimate the transient matrix \( C_{ij} \). We can write the simplified turbulent kinetic energy equation at a single grid cell \( i \) as

\[
\frac{dE_i}{dt} = -(\overline{u'w'}), \left( \frac{\partial U}{\partial z} \right)_i - (\overline{u'w'}), \left( \frac{\partial V}{\partial z} \right)_i + pc(i)(0.5C_{ai}aM^3)_i + \frac{g}{(\Theta_v)_i}(\overline{w'\Theta_v')}_i - (\varepsilon)_i, 
\]

where \( pc(i) \) has been defined before, \( M = \sqrt{U^2 + V^2} \), \( \Theta_v \) is the virtual potential temperature which can be calculated from the potential temperature and the moisture, and \( g \) is the acceleration due to gravity. When grid cell \( i \) is located within the plant canopy, it is the spatially averaged TKE equation (Kaimal and Finnigan, 1994), including the additional wake turbulence term \( (0.5C_{ai}aM^3) \), when the grid cell is located within the canopy. Otherwise it is the simplified TKE equation in the PBL.

Following Stull and Driedonks (1987), we assume that the turbulent kinetic energy \( E_{ij} \) between the non-adjacent levels \( i \) and \( j \) can be deduced from Eq. (32) by

\[
\frac{\Delta E_{ij}}{\Delta t} = -(\overline{u'w'}),_ij \left( \frac{\Delta V}{\Delta z} \right)_{ij} - (\overline{u'w'}),_ij \left( \frac{\Delta V}{\Delta z} \right)_{ij} + pc(i,j)(0.5C_{ai}aM^3)_ij + \frac{g}{(\Theta_v)_ij}(\overline{w'\Theta_v')}_ij - (\varepsilon)_{ij},
\]

where \( \Delta E_{ij} \) is the change in \( E_{ij} \) during time \( \Delta t \),

\[
\left( \frac{\Delta S}{\Delta z} \right)_{ij} = \frac{S_j - S_i}{z_j - z_i},
\]

\( S \) denotes \( U, V, (\Theta_v)_i = (\Theta_v(i) + \Theta_v(j))/2 \); the flux terms \((\overline{u'w'}),_ij, (\overline{v'w'}),_ij, \) and \((\overline{w'\Theta_v')}_ij\) are the
differences in the fluxes between the two levels $i$ and $j$; and \((0.5C_d aM^3)_{ij}\) is the wake turbulence energy between non-adjacent levels $i$ and $j$.

As suggested by Stull and Batsnicki (1993),

\[
Y_{ij} = \frac{1}{E_{ij}T_0} \frac{\Delta z E_{ij}}{\Delta t},
\]

where $T_0$ is the turbulence time scale. Moreover (see Stull and Driedonks, 1987),

\[
\frac{-\left(\overline{u'w'}\right)_{ij}}{E_{ij}T_0} = \left(\frac{\Delta U}{\Delta z}\right)_{ij},
\]

(35)

\[
\frac{-\left(\overline{v'w'}\right)_{ij}}{E_{ij}T_0} = \left(\frac{\Delta V}{\Delta z}\right)_{ij},
\]

(36)

\[
\frac{-\left(\overline{w'\theta'}\right)_{ij}}{E_{ij}T_0} = \frac{1}{R_{ic}} \left(\frac{\Delta \theta}{\Delta z}\right)_{ij},
\]

(37)

where the critical Richardson number $Ric = 0.21$, the dissipation parameter $D = 1.0$, and the turbulence time scale $T_0 = T_{0,\text{BL}} = 1000$ s for air flow in the PBL, and $T_0 = T_{0,C} = 10$ s is the canopy air flow.

We now parametrize the additional wake turbulence term \((0.5C_d aM^3)_{ij}/(E_{ij}T_0)\) based on Taylor’s scale arguments (Corrsin, 1963),

\[
L_w = \sigma_w T_w
\]

(39)

where $L_w$ is the single-point Eulerian length scale for vertical velocity, $\sigma_w$ is the standard deviation of $w$, and $T_w$ is the time scale for vertical velocity. As noted by Raupach (1989), for $z$ close to $h_c$, $L_w \approx h_c/3$. Thus

\[
E_{ij} = u'^2 + v'^2 + w'^2 \approx 3\sigma_w^2 \approx 3\left(\frac{L_w}{T_w}\right)^2 \approx \frac{h_c^2}{3* T_{0,C}^2}.
\]

(40)

where we have employed $T_w = T_{0,C}$. Therefore

\[
\frac{(0.5C_d aM^3)_{ij}}{E_{ij}T_0} = \frac{1.5C_d a_{ij} M^3_{ij} T_{0,C}}{h_c^2}.
\]

(41)

Since grid cells $i$ and $j$ can be located within and outside the plant canopy, and the formulas for the wake turbulence and dissipation terms are different for different combinations of levels $i$ and $j$, we have to distinguish the various cases below. First we recall the following: $T_{0,C} = 10$ s is the turbulence time scale for the air flow within the canopy, $T_{0,\text{BL}} = 1000$ s is the corresponding one in the PBL, and the height of the roughness sublayer is $3h_c$. Let $z_i$ and $z_j$ be the heights of the centers of the grid cells $i$ and $j$ from the ground. Assume $i \neq j$.

Case 1: $z_i \geq h_c$ and $z_j \geq h_c$, i.e. both grid cells are in the PBL. Following Stull’s parametrization scheme, we have

\[
Y_{ij} = \left(\frac{\Delta U}{\Delta z}\right)_{ij}^2 + \left(\frac{\Delta V}{\Delta z}\right)_{ij}^2 - \frac{g}{Ric} \left(\frac{\Delta \theta}{\Delta z}\right)_{ij} - \frac{D}{T_{0,\text{BL}}^2}.
\]

(42)
Case 2: \( z_i \leq h_c \) and \( z_j \leq h_c \), i.e. both grid cells are within the plant canopy. The existence of the canopy structure makes the turbulence within the canopy dissipate faster than that in the PBL. Moreover, it also creates wake turbulence. We have

\[
Y_{ij} = \left( \frac{\Delta U}{\Delta z} \right)_{ij}^2 + \left( \frac{\Delta V}{\Delta z} \right)_{ij}^2 - \frac{g}{R_i (\Theta_i)} \frac{\Delta \Theta_i}{\Delta z} \frac{1.5 C_{a,i} M_{ij}^3 T_{0,C}}{h_c^2}, \tag{43}
\]

where \( a_{ij} = (a(i) + a(j))/2 \) and \( M_{ij} = \left( \sqrt{U_i^2 + V_i^2} + \sqrt{U_j^2 + V_j^2} \right)/2 \).

Case 3: \( z_i \leq h_c \) and \( h \leq z_j \leq 3h_c \), i.e. one grid cell is located within the plant canopy and the other is above the canopy but within the roughness sublayer. Then

\[
Y_{ij} = \left( \frac{\Delta U}{\Delta z} \right)_{ij}^2 + \left( \frac{\Delta V}{\Delta z} \right)_{ij}^2 - \frac{g}{R_i (\Theta_i)} \frac{\Delta \Theta_i}{\Delta z} \frac{1.5 C_{a,i} M_{ij}^3 T_{0,C}}{h_c^2}, \tag{44}
\]

where \( M_{ij} = \left( \sqrt{U_i^2 + V_i^2} + \sqrt{U_j^2 + V_j^2} \right)/2 \), and \( U_i \) and \( V_i \) are the \( x, y \) components of mean wind vectors at the canopy height \( h \), since the wake turbulence from leaf drag exists only within the plant canopy. The dissipation term

\[
Dis = \frac{D h_c - z_i}{T_{0,C}^2 z_i - z_j} + \frac{D z_j - h_c}{T_{0,BL}^2 z_j - z_i}.
\]

Case 4: \( h \leq z_i \leq 3h_c \) and \( z_j \leq h_c \). This case is similar to case 3,

\[
Y_{ij} = \left( \frac{\Delta U}{\Delta z} \right)_{ij}^2 + \left( \frac{\Delta V}{\Delta z} \right)_{ij}^2 - \frac{g}{R_i (\Theta_i)} \frac{\Delta \Theta_i}{\Delta z} \frac{1.5 C_{a,i} M_{ij}^3 T_{0,C}}{h_c^2}, \tag{45}
\]

where \( M_{ij} = \left( \sqrt{U_i^2 + V_i^2} + \sqrt{U_j^2 + V_j^2} \right)/2 \), and

\[
Dis = \frac{D h_c - z_j}{T_{0,C}^2 z_j - z_i} + \frac{D z_i - h_c}{T_{0,BL}^2 z_i - z_j}.
\]

Case 5: \( z_i \geq 3h_c \) and \( z_j \leq h_c \). This case involves eddies within the canopy moving up to destinations above the roughness sublayer. Based on the definition of the roughness sublayer in which the presence of the canopy impinges directly on the character of the turbulence, in other words, the direct effects of the canopy-size eddies are confined below the roughness sublayer, the eddies generated within the canopy layer will not go above the roughness sublayer. We assume no mixing for this case:

\[
Y_{ij} = 0. \tag{46}
\]

Case 6: \( z_i \leq h_c \) and \( z_j \geq 3h_c \). This case involves eddies of the size of boundary layer depth which impinge down to destinations in the canopy layer. The mixing can not be zero, and we parametrize \( Y_{ij} \) as

\[
Y_{ij} = \left( \frac{\Delta U}{\Delta z} \right)_{ij}^2 + \left( \frac{\Delta V}{\Delta z} \right)_{ij}^2 - \frac{g}{R_i (\Theta_i)} \frac{\Delta \Theta_i}{\Delta z} \frac{D}{T_{0,BL} T_{0,C}}, \tag{47}
\]

Note that when one grid cell is within the canopy layer and the other is above the roughness sublayer, \( Y_{ij} \) is not symmetric.

We have covered all the cases when \( i \neq j \). In other words, the mixing potential \( Y_{ij} \) can be computed when \( i \neq j \).

We now turn to the determination of \( Y_{ii} \). In Stull and Driedonks (1987),

\[
Y_{ii} = \max(Y_{i,i-1}, Y_{i,i+1}) + Y_{ref}, \tag{48}
\]
with \( Y_{\text{ref}} = 1000 \) for their chosen uniform grid. Here, for our uneven grid, we have modified \( Y_{\text{ref}} = Y_{\text{den}} \Delta z_i \), where the new parameter \( Y_{\text{den}} \) is the reference mixing potential density (with unit \( m^{-1} \)). Hence
\[ Y_{ii} = \max(Y_{i,i-1}, Y_{i,i+1}) + Y_{\text{den}} \Delta z_i \]  
(49)

\( Y_{ii} \) increases with grid cell size \( \Delta z_i \) because a larger grid cell can accommodate more internal eddies. The reference mixing potential density is kept constant. From our numerical experiments, we suggest \( Y_{\text{den}} = 0.003 \).

The interference coefficient \( \gamma(k) \) of leaves for turbulent mixing, introduced by Stull (1990), measures the relative amount of obstruction to turbulent mixing. Stull (1990) divided the canopy into three layers, and \( \gamma(1) = 0.1, \gamma(2) = 0.3 \) and \( \gamma(3) = 0.7 \). In this study, \( \gamma(k) \) is regarded as a function of the normalized leaf area index in each grid cell \( k \) within the plant canopy. Otherwise, the interference coefficient is assumed to be zero. So we have
\[
\gamma(k) = \begin{cases} 
\frac{\|L_a(k-1) - L_a(k)\|}{L_a(0)}, & \text{if grid cell } k \text{ is within plant canopy} \\
0, & \text{if grid cell } k \text{ is outside plant canopy}
\end{cases}
\]  
(50)

where \( L_a(0) \) is the accumulated leaf area index at the ground surface. The mixing potential \( Y_{ij} \) is then modified as
\[
Y_{ij}^{\text{new}} = Y_{ij} \prod_{k=i}^{k=j-1} (1 - \gamma(k)) \text{ for } i < j,
\]  
(51)
\[
Y_{ij}^{\text{new}} = Y_{ij} \prod_{k=j}^{k=i-1} (1 - \gamma(k)) \text{ for } j < i.
\]  
(52)

To account for the influence of the different grid sizes, the adjusted mixing potential \( A_{ij} \) is calculated from
\[
A_{ij} = m_j Y_{ij}^{\text{new}},
\]  
(53)

where \( m_j \) is the air mass of the source grid cell \( j \). Defining \( \|A\| = \max_i \sum_j A_{ij} \), we obtain
\[
C_{ij} = A_{ij}/\|A\| \text{ if } i \neq j.
\]  
(54)

The transient coefficients are therefore known when \( i \neq j \). To preserve Eq. (4), we take
\[
C_{ii} = 1 - \sum_{j=1, j \neq i}^n C_{ij}.
\]  
(55)

Thus \( C_{ij} \) is now determined for all \( i \) and \( j \) in our coupled model.

With all variables known at the current time step, the transient matrix can now be computed. The variables and turbulence properties at the next time step can then be obtained by Eqs. (28)–(31). These iterations allow us to compute the solutions at any time.

Remark 1: From its definition, \( C_{ij} \) tends to the identity matrix as \( \Delta t \) goes to zero. This is because there is no transport of eddies to other cells in zero time.

Remark 2: Eqs. (28)–(31) are similar to those in Stull’s formulation. In his original scheme, the equations were tackled by a two-step procedure to advance a time step of \( \Delta t \). If we lump all the source terms to be \( g_i \), then Stull’s procedure gives

\begin{align*}
\text{Step 1: } & \hat{U}_i(t) = U_i(t) + \Delta t g_i(t), \quad (56) \\
\text{Step 2: } & U_i(t + \Delta t) = \sum_{j=1}^n C_{ij} \hat{U}_j(t), \quad (57)
\end{align*}
Thus, denoting the elements of the identity matrix $I_{ij}$, we have

\[ U_i(t + \Delta t) = \sum_{j=1}^{n} C_{ij}(U_j(t) + \Delta t g_j(t)) \]

(58)

\[ = \sum_{j=1}^{n} C_{ij}U_j(t) + \Delta t g_i(t) + \Delta t \sum_{j=1}^{n} (C_{ij} - I_{ij}) g_j(t). \]

(59)

As $\Delta t$ goes to zero, the last term on the right-hand side is smaller than the others in view of Remark 1. Provided $\Delta t$ is small enough (while keeping the size of the grid cell fixed), the contribution due to this last term is insignificant. Deleting such a term reduces the two-step procedure to our equations.

**Remark 3:** In our actual numerical computations, we employ the two-step procedure for easy comparison with Stull's result. Details of this procedure are deferred to the next section.

4. Numerical simulation

4.1. Observed data in the PBL and within the plant canopy

The coupled T-theory model was tested against experimental data collected in the Black Moshannon Forest, Pennsylvania, USA (latitude 40°59' N, longitude 78°6' W, elevation 625 m), on 30 May 1990.

For the PBL profile measurements, radiosondes with temperature, pressure and humidity sensors (AIR Inc., Boulder, CO) on pilot balloons, tackled with a recording theodolite, were periodically released from a flat forest clearing about 100 × 100 m, surrounded by trees 20 m high (mixed red oaks, white oaks, chestnut oaks, black oaks and a few maples). The within-canopy profile measurements were made at six levels, two above and four within the canopy, with a three-axis propeller anemometer, TFWOES hot film anemometers (Miller et al., 1989), fine wire (0.013 mm) thermocouples, coarse wire (0.8 mm) thermocouples (for mean temperature) and humidity sensors (Vaisala, Inc., model HMT14), on a 40 m high tower within the forest. The fast response instruments were recorded at a scanning rate of 29 Hz for periods of 30 min spanning the time of each run. The slow response instruments were recorded with a data logger (model 21X, Campbell Scientific, Logan, UT) at 10 s intervals. The radiation at the top of the forest and the leaf area index (LAI) were also measured (Wang et al., 1992). For a complete description of the experimental measurements, see Miller et al. (1995).

Figs. 1 and 2 show the potential temperature and mean wind speed profiles, respectively, interpreted from the...
periodic radiosonde data. Table 1 shows the depth of the PBL. Figs. 3–6 show the observed wind speed, momentum flux, potential temperature and sensible heat flux averaged over 4 min within and just above the forest.

As can be seen in Fig. 1, the potential temperature decreased with height in the surface layer, and was constant in the mixing layer and then increased in the entrainment zone. The magnitude of the potential temperature underwent a diurnal change. During the daytime the potential temperature in the mixing layer increased in the morning, reached a maximum value in the early afternoon, and then remained constant until the late afternoon (18:00).

Fig. 2 shows that the wind speed increased with height in the surface layer and was constant in the mixing layer. For the diurnal change, the wind speed in the mixing layer decreased in the morning, reached a minimum value around noon, and then remained almost constant throughout the afternoon. The observed data also show that in the morning of 30 May 1990, there was a local jet at the top of the boundary layer. There was a very strong convective mixing layer throughout the afternoon. The boundary layer depth suddenly increased from 1200 m at 12:00 to 1600 m at 13:00, and remained constant throughout the afternoon (see Table 1).

Fig. 3 shows a rapid decrease in wind speed from levels just above the plant canopy (25 m) to the upper part of the canopy (15 m height), which might have been due to the nonuniform tree heights, and a decrease in wind speed with height in the lower part of the canopy (0–10 m) at 10:00 on that day. This indicated a "bulge" in the deep canopy (Raupach et al., 1991), which was also observed in the Uriarra Forest by Denmead and Bradley (1987).

### Table 1

<table>
<thead>
<tr>
<th>Time (EST)</th>
<th>Obs. (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00</td>
<td>400</td>
</tr>
<tr>
<td>10:00</td>
<td>800</td>
</tr>
<tr>
<td>11:00</td>
<td>1000</td>
</tr>
<tr>
<td>12:00</td>
<td>1200</td>
</tr>
<tr>
<td>13:00</td>
<td>1600</td>
</tr>
<tr>
<td>16:33</td>
<td>1600</td>
</tr>
<tr>
<td>17:51</td>
<td>1600</td>
</tr>
<tr>
<td>18:54</td>
<td>1600</td>
</tr>
</tbody>
</table>
Fig. 4 shows a strong downward momentum flux in the upper part of the canopy, but within the deep canopy the downward turbulence momentum transfer was very weak. The wind speed and momentum flux profiles at levels where $dU/dz < 0$ indicate a counter-gradient momentum transfer. For this case, the traditional K-theory fails.

Fig. 5 indicates a sigmoid-shaped potential temperature profile within the canopy. For example, there is a local maximum potential temperature in the upper part of the canopy, and a global maximum potential temperature at the ground surface. The local maximum potential temperature corresponds to the level with maximum foliage area volume density (see below). Due to the coarse resolution in the field measurements, this local maximum potential temperature in the upper canopy might not be easily detectable, but it was observed in the Uriarra Forest by Denmead and Bradley (1985). Since the canopy was sparse, the global maximum potential temperature was at the ground surface due to the larger proportion of solar radiation penetrating to the ground surface.

Fig. 4. Observed momentum flux ($u'w'$) within and just above Black Moshannon Forest, PA, 30 May 1990. The large negative flux at the canopy height ($h = 20$ m) indicates a large downward momentum transfer.
Fig. 5. Observed potential temperature within and just above Black Moshannon Forest, PA, 30 May 1990.

Fig. 6 shows an upward transfer of sensible heat within the whole canopy layer, but the magnitude is very small in the deep canopy. The potential temperature and sensible heat flux at levels where $\frac{d\theta}{dz} > 0$ and $\overline{w'\theta'} > 0$ indicate a counter-gradient sensible heat transfer.

With the leaf area index data collected in the Black Moshannon Forest on 30 May 1990, Yang et al. (1993) formulated the downward accumulated leaf area index $L_a(z)$ as the Weibull cumulative distribution function, and the leaf area density can be easily obtained (see Fig. 7). With the solar radiation data collected at the top and within the canopy, Yang et al. (1993) also formulated the solar transmission within the canopy as a function of the accumulated leaf area index $L_a(z)$,

$$ r_{se}(z) = r_a e^{-0.64L_a(z)}. $$

The solar radiation absorption can be estimated by taking the derivative of Eq. (60).

Fig. 8 shows the distribution of solar radiation at the top of the canopy, and Fig. 9 shows the calculated transmission and absorption of solar radiation by leaves. Based on this distribution of solar radiation absorption by the canopy, the heat and moisture exchanges between the leaf surface and the surrounding air can be
Fig. 7. Accumulated leaf area index (top) and foliage area volume density (bottom) in Black Moshannon Forest, PA, 30 May 1990.

Fig. 8. Observed solar radiation at the top of Black Moshannon Forest, PA, 30 May 1990.

Fig. 9. Calculated solar irradiance (top) and absorption (bottom) for Black Moshannon Forest, PA, 30 May 1990.
obtained by the Bowen ratio method (see Fig. 10). These form the inputs for running the coupled T-theory model.

4.2. Simulation procedure

As shown in Table 1, the maximum observed boundary layer depth is about 1600 m, above which there is no scalar transfer. We therefore created a computation domain of height 1900 m from the ground, and the whole domain was divided into 45 uneven grid cells. A finer grid size was employed within and just above the canopy, and a coarser grid at higher elevations. The precise grid sizes are given in Fig. 11.

The physical constant parameters required for the implementation of the coupled model are given in Table 2. The other input data including the accumulated leaf area index, foliage area volume density, solar radiation at the top of the canopy, solar radiation absorption by leaves, and the sensible and latent heat fluxes at the leaf surface are shown in Figs. 7–10. Using the wind speeds, and the potential temperature data collected at 9:00 as initial conditions, the coupled T-theory model was run with time steps of 1 s for a period of 12 h.
Table 2
Input parameters of the coupled and original T-theory models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_c$</td>
<td>0.21</td>
<td>Critical Richardson number</td>
</tr>
<tr>
<td>$T_{oc}$</td>
<td>10</td>
<td>Turbulence time scale of canopy flow</td>
</tr>
<tr>
<td>$T_{oBL}$</td>
<td>1000</td>
<td>Turbulence time scale of atmospheric boundary layer</td>
</tr>
<tr>
<td>$D$</td>
<td>1</td>
<td>Dissipation</td>
</tr>
<tr>
<td>$Y_{den}$</td>
<td>0.003</td>
<td>Reference mixing potential density ($m^{-3}$)</td>
</tr>
<tr>
<td>$C_{dl}$</td>
<td>0.2</td>
<td>Leaf drag coefficient including the shelter effect</td>
</tr>
</tbody>
</table>

The upper boundary of our simulation domain was just above the highest turbulent boundary layer depth (1900 m); zero turbulence fluxes for upper boundary layer conditions were assumed. The lower boundary layer was the ground surface for the coupled T-theory model.

For comparison, Stull’s original T-theory was run using the same physical parameters, initial conditions and time steps as described above. The upper boundary was the same as that in the coupled T-theory model, but the lower boundary was the top of the canopy. This lower boundary conditions should include the integrated effects of the whole plant canopy. The grid cell sizes are shown in Table 3.

For the coupled T-theory model run, the sensible heat flux $F_{o,\theta}$ and latent heat flux $F_{o,q}$ at the ground surface were calculated based on the solar radiation penetrating through the canopy and absorbed by the soil, with 10% of it for downward soil heat flux (Monteith and Unsworth, 1990), and the rest for upward sensible and latent heat fluxes at the ground surface. The proportions of sensible and latent heat fluxes were obtained by the Bowen ratio method (see Fig. 12). A typical value of the Bowen ratio of 0.5 was applied for the forest. An averaged absorption coefficient for soil of 0.85 (Irons et al., 1989) was used.

The momentum flux at the ground surface was calculated from a bulk transfer relation (Taylor, 1916) based on the wind speed at the lowest grid point. According to the drag law, we can employ the momentum flux as (Stull, 1991)

$$F_{o,u} = C_{du}\left(U_1^2 + V_1^2\right)^{1/2} U_1,$$

$$F_{o,v} = C_{dv}\left(U_1^2 + V_1^2\right)^{1/2} V_1,$$

where $C_{du}$ is the drag coefficient of the ground roughness, which is a function of the stability of the flow. Here,
for simplicity, a constant drag coefficient at neutral flow (Louis, 1979) was used during the period of simulation with the assumption that the sensor is at the top of the first grid. Hence

\[
C_{ds} = \left( \frac{0.4}{\ln(z_1/z_0)} \right)^2,
\]

(63)

where \( z_0 = 0.001 \) m, leading to \( C_{ds} = 0.0075 \) in the coupled T-theory model.

For the original T-theory model run, based on the observed sensible heat flux at the top of the canopy (Fig. 13), a half-sine wave pattern of sensible heat flux at the ground surface was applied. The latent heat flux was parametrized by the Bowen ratio method. For the momentum flux at the top of the canopy, we used the same strategy as in the coupled T-theory model, but with the roughness length for a vegetated surface. With \( z_0 = 0.08 h_c \) for forests (Kaimal and Finnigan, 1994), and \( h_c = 20 \) m, then \( z_0 = 1.6 \) m, since we employed \( z_1 = 50 \) m and \( C_{ds} = 0.0135 \).
4.3. Results and discussion

First we analyze the results for the whole PBL. Figs. 14–17 show the simulated potential temperature, sensible heat flux, wind speed, and momentum flux profiles, respectively, in the PBL using the coupled T-theory model. Figs. 18–21 show the simulation results using the original T-theory model.

A comparison of the simulated potential temperature (Figs. 14 and 18) and wind speed (Figs. 16 and 20) profiles with the observation data shows that there is reasonable agreement between the general patterns of diurnal changes predicted by both the coupled and original T-theory models. Figs. 15, 19 and Figs. 17, 21 also show very similar results for the sensible heat flux and momentum flux predicted by the two models. These results demonstrate that the models have the same capability to simulate turbulent mixing in the PBL. However, in the simulations of wind speed and potential temperature by the coupled T-theory model there is a clearer surface layer (less than 120–150 m high), than in those of the original T-theory model, due to the finer grid sizes used in the former model (see Fig. 11 and Table 3). Our coupled T-theory not only accounts for nonlocal turbulent mixing throughout the whole PBL, but it also simulates nonlocal turbulent mixing by canopy-size
Fig. 16. Simulated mean wind speed over Black Moshannon Forest, PA, 30 May 1990, using the coupled T-theory model.

Fig. 17. Simulated momentum flux over Black Moshannon Forest, PA, 30 May 1990, using the coupled T-theory model.

Fig. 18. Simulated potential temperature for Black Moshannon Forest, PA, 30 May 1990, using the original T-theory model.
Fig. 19. Simulated sensible heat flux for Black Moshannon Forest, PA, 30 May 1990, using the original T-theory model.

Fig. 20. Simulated wind speed for Black Moshannon Forest, PA, 30 May 1990, using the original T-theory model.

Fig. 21. Simulated momentum flux for Black Moshannon Forest, PA, 30 May 1990, using the original T-theory model.
### Table 4
Comparison of observed PBL depths with the results of the original coupled T-theory model simulations, 30 May 1990

<table>
<thead>
<tr>
<th>Time</th>
<th>Obs. (m)</th>
<th>Original T (m)</th>
<th>Coupled T (m)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>700</td>
<td>800</td>
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<td>10:00</td>
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<td>16:33</td>
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<td>1600</td>
</tr>
</tbody>
</table>

### Table 5
Comparison of observed potential temperature in the mixing layer with the results of the original and coupled T-theory model simulations, 30 May 1990

<table>
<thead>
<tr>
<th>Time</th>
<th>Obs. (K)</th>
<th>Original T (K)</th>
<th>Coupled T (K)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>282.8</td>
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<td>10:00</td>
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<td>284.0</td>
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<td>285.5</td>
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<td>289.0</td>
<td>289.0</td>
<td>289.1</td>
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</table>

### Table 6
Comparison of observed wind speeds in the mixing layer with the results of the original and coupled T-theory model simulations, 30 May 1990

<table>
<thead>
<tr>
<th>Time</th>
<th>Obs. (m s(^{-1}))</th>
<th>Original T (m s(^{-1}))</th>
<th>Coupled T (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
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<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>10:00</td>
<td>14.0</td>
<td>8.0</td>
<td>6.5</td>
</tr>
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<td>11:00</td>
<td>12.5</td>
<td>8.0</td>
<td>7.0</td>
</tr>
<tr>
<td>13:00</td>
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<td>8.5</td>
</tr>
<tr>
<td>18:00</td>
<td>10.2</td>
<td>8.5</td>
<td>8.0</td>
</tr>
</tbody>
</table>

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**Fig. 22.** Simulated mean wind speed within and just above Black Moshannon Forest, PA, 30 May 1990, using the coupled T-theory model.
eddies and turbulent boundary-layer-size eddies in the surface layer. A finer grid size in the original T-theory model might not reproduce such refined features because the effect of canopy air flow was lumped as a surface roughness length parameter.

Tables 4–6 compare the PBL depths, the potential temperatures and wind speeds in the mixing layer of the PBL between the observed data and simulation results from the original and coupled T-theory models. Both models predict potential temperature values close to those observed, but the predictions of the PBL depth and the wind speed are lower than those observed, indicating that the two models tend to underestimate the intensity of turbulence in the PBL. One possible reason is that both models are one-dimensional, and ignore horizontal turbulent transport. The observation data show a strong jet at the top of the PBL on the day of observations (see the wind profiles in Fig. 2), indicating a heterogeneity of horizontal turbulent air flow. A three-dimensional T-theory model that takes into account horizontal convection may improve the simulation results. This will be left for further study.

An important aspect of the coupled T-theory model is its ability to simulate turbulent air flow within and just above the canopy. Figs. 22–25 show the coupled T-theory model simulations of mean wind speed, momentum
flux, mean potential temperature and sensible heat flux profiles within and just above the canopy. In Fig. 22 the simulated mean wind speed profile just above the canopy is logarithmic and then decreases rapidly at the top of the canopy, reaching a minimum value in the upper canopy with a “bulge” in the lower canopy (Raupach et al., 1991). In Fig. 23 the momentum flux profile (shear stress) exhibits a constant downward transfer just above the canopy; the downward transfer attenuates rapidly as \( z \) decreases within the canopy, and becomes very weak within the deep canopy. These properties of the simulated wind speed and momentum flux profiles are the same as those observed in the field measurements (see Section 4.1), especially those indicating a counter-gradient momentum transfer at levels where \( dU/dz < 0 \). This demonstrates that the coupled T-theory model can generate wind and momentum profiles that are in reasonable agreement with field observations, and in particular can predict the counter-gradient momentum transfer in which the traditional K-theory fails.

Fig. 24 shows the simulated potential temperature, which decreases with height just above the canopy. This is expected since solar radiation absorption by the canopy elements and the ground surface warms the surrounding air and results in a decrease in potential temperature with height in the surface layer. The simulated potential
temperature profile within the canopy shows (1) a local maximum value at a height of 15 m, coinciding with the level of maximum leaf area volume density (see Fig. 7) corresponding to the maximum solar radiation absorption by foliage; and (2) a global maximum temperature at the ground surface. Fig. 25 exhibits a constant upward sensible heat transfer just above the plant canopy and then decreases rapidly in the upper and middle part of the canopy, and finally becomes very weak in the deep canopy. The weak sensible heat transfer in the deep canopy is due to the obstruction in the upper part of canopy, resulting in less heat being transferred from the ground surface to the upper part and even out of the canopy. All of these findings indicate a counter-gradient sensible heat transfer in the region where \( \frac{d\theta}{dz} > 0 \). These simulation results are in good agreement with the observed potential temperature and sensible heat flux, as discussed in Section 4.1.

Counter-gradient scalar (momentum, heat, water vapor and \( \text{CO}_2 \)) transfers have also been reported from direct experiments in the Uriarra Forest (Denmead and Bradley, 1985, 1987), the Camp Borden forest (Gao et al., 1989), and in a pine forest (Bergström and Högström, 1989). As we know, the physical background of counter-gradient scalar transfer is due to the domination of larger and cooler eddies just above the canopy impinging down into the canopy, and larger and warmer eddies within the canopy sweeping out of the canopy, resulting in downward momentum transfer and upward sensible heat transfer throughout the canopy layer. Our simulation results demonstrate that the coupled T-theory model is capable of predicting the turbulence properties well.

A comparison of the modeled potential temperature and sensible heat flux (Fig. 26), and wind speed and momentum flux (Fig. 27) with field measurements shows that there are some deviations of the modeled potential temperature and wind speed profiles from the observed data. The coupled T-theory model predicts a stronger sigmoid-shaped potential temperature profile and a more rapid wind shear at the top of the canopy than those observed. It seems that there are excessive effects due to the canopy structure on the wind speed and potential temperature. One reason for this could be the spatial variability in the foliage area volume density. The measured foliage area volume density is the averaged value of nine measurements at different locations, and our wind speed and potential temperature were measured at only one location, so that the latter measurements may include the effects of the local canopy structure. As pointed out by Wang et al. (1992), the foliage area volume density exhibits large spatial variability, especially in the upper part of the canopy. Another possible reason is that in the calculation of solar radiation absorbed by leaves, we simply approximated it from the light interception. This could have led to an overestimate of the absorbed solar radiation, since some of it is scattered by leaves of the canopy.
5. Conclusions

For turbulent air flow within the plant canopy, due to the effects of leaf drag on air flow, strong wind shear takes place at the top. Cooler and faster large eddies impinge down into the canopy, and warmer and slower large eddies sweep out of the canopy. Both effects result in nonlocal turbulent air flow within and just above the canopy. The counter-gradient turbulent air flow within the canopy, which the traditional K-theory fails to explain, is the result of this nonlocal mixing. Based on Stull's nonlocal (transilient) theory, which considers turbulent mixing as a convective rather than a diffusion process, we have constructed a coupled transilient-theory (T-theory) model for turbulent flow in the canopy and the PBL. This can be viewed as an extension of Stull's original T-theory in the PBL to the canopy layer.

Since in the T-theory, different turbulence mechanisms of canopy flow can be accounted for by different parametrizations of the transilient matrix, one can simulate the influence of the canopy structure on turbulent air flow within and above it. This study provides a physically based method for calculating the surface flux transfer over the vegetation surface in which the traditional K-theory fails.

A comparison of the results with data collected from the Black Moshannon Forest showed that the coupled T-theory model can not only simulate turbulent mixing in the PBL as well as the original T-theory model, but it can also predict the properties of turbulence within and just above the canopy reasonably well, especially the counter-gradient turbulent transport.

This coupled T-theory model is one-dimensional, but future work may involve its extension to a three-dimensional model in order to account for the horizontal turbulent transport that has so far been ignored.

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Appendix A. List of symbols

\[ A \] Adjusted mixing potential matrix for uneven grid spacings
\[ \| A \| \] Norm of \( A \)
\[ a \] Leaf area density (m\(^{-1}\))
\[ C \] Transilience matrix
\[ C_{os} \] Surface drag coefficient
\[ C_{ol} \] Leaf drag coefficient of plant canopy, including shelter factor
\[ C_p \] Specific heat of air at constant pressure (J kg\(^{-1}\) K\(^{-1}\))
\[ C_v \] Specific heat of water vapor (J kg\(^{-1}\) K\(^{-1}\))
\[ d \] Zero-plane displacement (m)
\[ D \] Dimensionless dissipation parameter
\[ E \] Turbulent kinetic energy per unit mass (J m\(^{-1}\))
\[ E_{ij} \] Turbulent kinetic energy between grid cells \( i \) and \( j \)
\[ F_k \] Specific turbulent flux at level \( k \)
\[ F_{0u} \] \( u'w' \) at ground surface (m\(^2\) s\(^{-2}\))
\[ F_{0v} \] \( v'w' \) at ground surface (m\(^2\) s\(^{-2}\))
$F_{0\theta}$: $\theta' w$ at ground surface (km s$^{-1}$)
$F_{0q}$: $q' w$ at ground surface (g kg$^{-1}$ ms$^{-1}$)
$h_c$: Plant canopy height (m)
$i,j,k$: Grid cell indices
$L$: Leaf area index (m$^2$ m$^{-2}$)
$L_a$: Accumulated downward leaf area index (m$^2$ m$^{-2}$)
$L_w$: Turbulence length scale in vertical direction
$m$: Air mass (kg)
$M$: Mean wind speed, $\sqrt{U^2 + V^2}$ (m s$^{-1}$)
$q$: Mean specific humidity (g kg$^{-1}$)
$q'$: Specific humidity departure from mean value (g kg$^{-1}$)
$q' w'$: Latent heat flux (g kg$^{-1}$ ms$^{-1}$)
$r_s$: Solar radiation at the top of the canopy (W m$^{-2}$)
$r_{ac}$: Solar radiation within the canopy (W m$^{-2}$)
$r_{al}$: Solar radiation absorbed by foliage per unit volume (W m$^{-3}$)
$R_e$: Dimensionless critical Richardson number in TTT
$S$: Specific property of air flow
$T_{0,c}$: Turbulence time scale within canopy (s)
$T_{0,BL}$: Turbulence time scale in atmospheric boundary layer (s)
$T_w$: Turbulence time scale in vertical direction (s)
$t$: Time (s)
$\Delta t$: Time interval (s)
$U$: $x$ component of mean wind vector (m s$^{-1}$)
$U_k$: $x$ component of geostrophic wind vector (m s$^{-1}$)
$u'$: Wind departure from $x$ component of mean wind vector (m s$^{-1}$)
$u' w'$: $x$ component of vertical momentum flux (m$^2$ s$^{-2}$)
$V$: $y$ component of mean wind vector (m s$^{-1}$)
$V_k$: $y$ component of geostrophic wind vector (m s$^{-1}$)
$v'$: Wind departure from $y$ component of mean wind vector (m s$^{-1}$)
$v' w'$: $y$ component of vertical momentum flux (m$^2$ s$^{-2}$)
$W$: $z$ (vertical) component of mean wind vector (m s$^{-1}$)
$w'$: Wind departure from $z$ component of mean wind vector (m s$^{-1}$)
$Y$: Mixing potential matrix
$||Y||$: Norm of $Y$
$Y_{den}$: Reference mixing potential density (m$^{-3}$)
$z$: Height (m)
$z_0$: Roughness length (m)
$z_i$: Height at the central point of grid cell $i$ (m)
$\Delta z_i$: Thickness of grid cell $i = z_i - z_{i-1}$ (m)
$\rho$: Air mass volume density (km$^{-3}$)
$\beta$: Bowen ratio of sensible to latent heat
$\gamma(k)$: Interference coefficient in grid cell $k$
$\Delta_j$: Difference over time $\Delta t$ (s)
$\varepsilon$: Dissipation term in KTE
$\Theta_k$: Mean virtual potential temperature (K)
$\theta'_i$: Virtual potential temperature departure from mean value (K)
\( \theta' w' \) Sensible heat flux (K m s\(^{-1}\))

\( \phi \) Latitude (deg)

\( \sigma_w \) Turbulence intensity in vertical direction

References


