Fluid mechanics is the discipline within the broad field of applied mechanics concerned with the behavior of liquids and gases at rest or in motion. This field of mechanics obviously encompasses a vast array of problems that may vary from the study of blood flow in the capillaries (which are only a few microns in diameter) to the flow of crude oil across Alaska through an 800-mile-long, 4-ft-diameter pipe. Fluid mechanics principles are needed to explain why airplanes are made streamlined with smooth surfaces for the most efficient flight, whereas golf balls are made with rough surfaces (dimpled) to increase their efficiency. Numerous interesting questions can be answered by using relatively simple fluid mechanics ideas. For example:

- How can a rocket generate thrust without having any air to push against in outer space?
- Why can't you hear a supersonic airplane until it has gone past you?
- How can a river flow downstream with a significant velocity even though the slope of the surface is so small that it could not be detected with an ordinary level?
- How can information obtained from model airplanes be used to design the real thing?
- Why does a stream of water from a faucet sometimes appear to have a smooth surface, but sometimes a rough surface?
- How much greater gas mileage can be obtained by improved aerodynamic design of cars and trucks?

The list of applications and questions goes on and on—but you get the point; fluid mechanics is a very important, practical subject. It is very likely that during your career as an engineer you will be involved in the analysis and design of systems that require a good understanding of fluid mechanics. It is hoped that this introductory text will provide a sound foundation of the fundamental aspects of fluid mechanics.
1.1 Some Characteristics of Fluids

One of the first questions we need to explore is—what is a fluid? Or we might ask—what is the difference between a solid and a fluid? We have a general, vague idea of the difference. A solid is “hard” and not easily deformed, whereas a fluid is “soft” and is easily deformed (we can readily move through air). Although quite descriptive, these casual observations of the differences between solids and fluids are not very satisfactory from a scientific or engineering point of view. A closer look at the molecular structure of materials reveals that matter that we commonly think of as a solid (steel, concrete, etc.) has densely spaced molecules with large intermolecular cohesive forces that allow the solid to maintain its shape, and to not be easily deformed. However, for matter that we normally think of as a liquid (water, oil, etc.), the molecules are spaced farther apart, the intermolecular forces are smaller than for solids, and the molecules have more freedom of movement. Thus, liquids can be easily deformed (but not easily compressed) and can be poured into containers or forced through a tube. Gases (air, oxygen, etc.) have even greater molecular spacing and freedom of motion with negligible cohesive intermolecular forces and as a consequence are easily deformed (and compressed) and will completely fill the volume of any container in which they are placed.

Although the differences between solids and fluids can be explained qualitatively on the basis of molecular structure, a more specific distinction is based on how they deform under the action of an external load. Specifically, a fluid is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude. A shearing stress (force per unit area) is created whenever a tangential force acts on a surface. When common solids such as steel or other metals are acted on by a shearing stress, they will initially deform (usually a very small deformation), but they will not continuously deform (flow). However, common fluids such as water, oil, and air satisfy the definition of a fluid—that is, they will flow when acted on by a shearing stress. Some materials, such as slurries, tar, putty, toothpaste, and so on, are not easily classified since they will behave as a solid if the applied shearing stress is small, but if the stress exceeds some critical value, the substance will flow. The study of such materials is called rheology and does not fall within the province of classical fluid mechanics. Thus, all the fluids we will be concerned with in this text will conform to the definition of a fluid given previously.

Although the molecular structure of fluids is important in distinguishing one fluid from another, it is not possible to study the behavior of individual molecules when trying to describe the behavior of fluids at rest or in motion. Rather, we characterize the behavior by considering the average, or macroscopic, value of the quantity of interest, where the average is evaluated over a small volume containing a large number of molecules. Thus, when we say that the velocity at a certain point in a fluid is so much, we are really indicating the average velocity of the molecules in a small volume surrounding the point. The volume is small compared with the physical dimensions of the system of interest, but large compared with the average distance between molecules. Is this a reasonable way to describe the behavior of a fluid? The answer is generally yes, since the spacing between molecules is typically very small. For gases at normal pressures and temperatures, the spacing is on the order of $10^{-6}$ mm, and for liquids it is on the order of $10^{-7}$ mm. The number of molecules per cubic millimeter is on the order of $10^{18}$ for gases and $10^{21}$ for liquids. It is thus clear that the number of molecules in a very tiny volume is huge and the idea of using average values taken over this volume is certainly reasonable. We thus assume that all the fluid characteristics we are interested in (pressure, velocity, etc.) vary continuously throughout the fluid—that is, we treat the fluid as a continuum. This concept will certainly be valid for all the circumstances considered in this text. One area of fluid mechanics for which the continuum concept breaks down is in the study of rarefied gases such as would be encountered at very high altitudes. In this case the spacing between air molecules can become large and the continuum concept is no longer acceptable.
1.2 Dimensions, Dimensional Homogeneity, and Units

Since in our study of fluid mechanics we will be dealing with a variety of fluid characteristics, it is necessary to develop a system for describing these characteristics both qualitatively and quantitatively. The qualitative aspect serves to identify the nature, or type, of the characteristics (such as length, time, stress, and velocity), whereas the quantitative aspect provides a numerical measure of the characteristics. The quantitative description requires both a number and a standard by which various quantities can be compared. A standard for length might be a meter or foot, for time an hour or second, and for mass a slug or kilogram. Such standards are called units, and several systems of units are in common use as described in the following section. The qualitative description is conveniently given in terms of certain primary quantities, such as length, L, time, T, mass, M, and temperature, Θ. These primary quantities can then be used to provide a qualitative description of any other secondary quantity: for example, area \( \equiv L^2 \), velocity \( \equiv LT^{-1} \), density \( \equiv ML^{-3} \), and so on, where the symbol \( \equiv \) is used to indicate the dimensions of the secondary quantity in terms of the primary quantities. Thus, to describe qualitatively a velocity, \( V \), we would write

\[
V \equiv LT^{-1}
\]

and say that "the dimensions of a velocity equal length divided by time." The primary quantities are also referred to as basic dimensions.

For a wide variety of problems involving fluid mechanics, only the three basic dimensions, \( L, T, \) and \( M \) are required. Alternatively, \( L, T, \) and \( F \) could be used, where \( F \) is the basic dimension of force. Since Newton’s law states that force is equal to mass times acceleration, it follows that \( F \equiv MLT^{-2} \) or \( M \equiv FL^{-1}T^2 \). Thus, secondary quantities expressed in terms of \( M \) can be expressed in terms of \( F \) through the relationship above. For example, stress, \( \sigma \), is a force per unit area, so that \( \sigma \equiv FL^{-2} \), but an equivalent dimensional equation is \( \sigma \equiv ML^{-1}T^{-2} \). Table 1.1 provides a list of dimensions for a number of common physical quantities.

All theoretically derived equations are dimensionally homogeneous—that is, the dimensions of the left side of the equation must be the same as those on the right side, and all additive separate terms must have the same dimensions. We accept as a fundamental premise that all equations describing physical phenomena must be dimensionally homogeneous. If this were not true, we would be attempting to equate or add unlike physical quantities, which would not make sense. For example, the equation for the velocity, \( V \), of a uniformly accelerated body is

\[
V = V_0 + at
\]

where \( V_0 \) is the initial velocity, \( a \) the acceleration, and \( t \) the time interval. In terms of dimensions the equation is

\[
LT^{-1} = LT^{-1} + LT^{-1}
\]

and thus Eq. 1.1 is dimensionally homogeneous.

Some equations that are known to be valid contain constants having dimensions. The equation for the distance, \( d \), traveled by a freely falling body can be written as

\[
d = 16.1t^2
\]

and a check of the dimensions reveals that the constant must have the dimensions of \( LT^{-2} \) if the equation is to be dimensionally homogeneous. Actually, Eq. 1.2 is a special form of the well-known equation from physics for freely falling bodies,

\[
d = \frac{gt^2}{2}
\]
in which \( g \) is the acceleration of gravity. Equation 1.3 is dimensionally homogeneous and valid in any system of units. For \( g = 32.2 \text{ ft/s}^2 \) the equation reduces to Eq. 1.2 and thus Eq. 1.2 is valid only for the system of units using feet and seconds. Equations that are restricted to a particular system of units can be denoted as restricted homogeneous equations, as opposed to equations valid in any system of units, which are general homogeneous equations. The preceding discussion indicates one rather elementary, but important, use of the concept of dimensions: the determination of one aspect of the generality of a given equation simply based on a consideration of the dimensions of the various terms in the equation. The concept of dimensions also forms the basis for the powerful tool of dimensional analysis, which is considered in detail in Chapter 7.

### Table 1.1

<table>
<thead>
<tr>
<th>Dimensions Associated with Common Physical Quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( FL ) System</td>
</tr>
<tr>
<td>Acceleration</td>
</tr>
<tr>
<td>Angle</td>
</tr>
<tr>
<td>Angular acceleration</td>
</tr>
<tr>
<td>Angular velocity</td>
</tr>
<tr>
<td>Area</td>
</tr>
<tr>
<td>Density</td>
</tr>
<tr>
<td>Energy</td>
</tr>
<tr>
<td>Force</td>
</tr>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td>Heat</td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Mass</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>Moment of a force</td>
</tr>
<tr>
<td>Moment of inertia (area)</td>
</tr>
<tr>
<td>Moment of inertia (mass)</td>
</tr>
<tr>
<td>Momentum</td>
</tr>
<tr>
<td>Power</td>
</tr>
<tr>
<td>Pressure</td>
</tr>
<tr>
<td>Specific heat</td>
</tr>
<tr>
<td>Specific weight</td>
</tr>
<tr>
<td>Strain</td>
</tr>
<tr>
<td>Stress</td>
</tr>
<tr>
<td>Surface tension</td>
</tr>
<tr>
<td>Temperature</td>
</tr>
<tr>
<td>Time</td>
</tr>
<tr>
<td>Torque</td>
</tr>
<tr>
<td>Velocity</td>
</tr>
<tr>
<td>Viscosity (dynamic)</td>
</tr>
<tr>
<td>Viscosity (kinematic)</td>
</tr>
<tr>
<td>Volume</td>
</tr>
<tr>
<td>Work</td>
</tr>
</tbody>
</table>
A commonly used equation for determining the volume rate of flow, \( Q \), of a liquid through an orifice located in the side of a tank is

\[
Q = 0.61 \ A \sqrt{2gh}
\]

where \( A \) is the area of the orifice, \( g \) is the acceleration of gravity, and \( h \) is the height of the liquid above the orifice. Investigate the dimensional homogeneity of this formula.

**Solution**

The dimensions of the various terms in the equation are \( Q = \text{volume/time} = L^3 T^{-1} \), \( A = \text{area} = L^2 \), \( g = \text{acceleration of gravity} = LT^{-2} \), and \( h = \text{height} = L \)

These terms, when substituted into the equation, yield the dimensional form:

\[
(L^3 T^{-1}) = (0.61)(L^2)(\sqrt{2})(LT^{-2})^{1/2}(L)^{1/2}
\]

or

\[
(L^3 T^{-1}) = [(0.61)\sqrt{2}](L^3 T^{-1})
\]

It is clear from this result that the equation is dimensionally homogeneous (both sides of the formula have the same dimensions of \( L^3 T^{-1} \)), and the numbers (0.61 and \( \sqrt{2} \)) are dimensionless.

If we were going to use this relationship repeatedly we might be tempted to simplify it by replacing \( g \) with its standard value of 32.2 ft/s\(^2\) and rewriting the formula as

\[
Q = 4.90 \ A \sqrt{h}
\]

A quick check of the dimensions reveals that

\[
L^3 T^{-1} = (4.90)(L^{5/2})
\]

and, therefore, the equation expressed as Eq. 1 can only be dimensionally correct if the number 4.90 has the dimensions of \( L^{1/2} T^{-1} \). Whenever a number appearing in an equation or formula has dimensions, it means that the specific value of the number will depend on the system of units used. Thus, for the case being considered with feet and seconds used as units, the number 4.90 has units of ft\(^{1/2}\)/s. Equation 1 will only give the correct value for \( Q \) (in ft\(^3\)/s) when \( A \) is expressed in square feet and \( h \) in feet. Thus, Eq. 1 is a *restricted* homogeneous equation, whereas the original equation is a *general* homogeneous equation that would be valid for any consistent system of units. A quick check of the dimensions of the various terms in an equation is a useful practice and will often be helpful in eliminating errors—that is, as noted previously, all physically meaningful equations must be dimensionally homogeneous. We have briefly alluded to units in this example, and this important topic will be considered in more detail in the next section.

### 1.2.1 Systems of Units

In addition to the qualitative description of the various quantities of interest, it is generally necessary to have a quantitative measure of any given quantity. For example, if we measure the width of this page in the book and say that it is 10 units wide, the statement has no meaning until the unit of length is defined. If we indicate that the unit of length is a meter, and define the meter as some standard length, a unit system for length has been established.
(and a numerical value can be given to the page width). In addition to length, a unit must be established for each of the remaining basic quantities (force, mass, time, and temperature). There are several systems of units in use and we shall consider three systems that are commonly used in engineering.

**British Gravitational (BG) System.** In the BG system the unit of length is the foot (ft), the time unit is the second (s), the force unit is the pound (lb), and the temperature unit is the degree Fahrenheit (°F), or the absolute temperature unit is the degree Rankine (°R), where

\[ °R = °F + 459.67 \]

The mass unit, called the *slug*, is defined from Newton’s second law (force = mass \( \times \) acceleration) as

\[ 1 \text{ lb} = (1 \text{ slug})(1 \text{ ft/s}^2) \]

This relationship indicates that a 1-lb force acting on a mass of 1 slug will give the mass an acceleration of 1 ft/s^2.

The weight, \( W \) (which is the force due to gravity, \( g \)) of a mass, \( m \), is given by the equation

\[ W = mg \]

and in BG units

\[ W \text{(lb)} = m \text{ (slugs)} \times g \text{ (ft/s}^2) \]

Since the earth’s standard gravity is taken as \( g = 32.174 \text{ ft/s}^2 \) (commonly approximated as 32.2 ft/s^2), it follows that a mass of 1 slug weighs 32.2 lb under standard gravity.

**International System (SI).** In 1960 the Eleventh General Conference on Weights and Measures, the international organization responsible for maintaining precise uniform standards of measurements, formally adopted the *International System of Units* as the international standard. This system, commonly termed SI, has been widely adopted worldwide and is widely used (although certainly not exclusively) in the United States. It is expected that the long-term trend will be for all countries to accept SI as the accepted standard and it is imperative that engineering students become familiar with this system. In SI the unit of length is the meter (m), the time unit is the second (s), the mass unit is the kilogram (kg), and the temperature unit is the kelvin (K). Note that there is no degree symbol used when expressing a temperature in kelvin units. The Kelvin temperature scale is an absolute scale and is related to the Celsius (centigrade) scale (°C) through the relationship

\[ K = °C + 273.15 \]

Although the Celsius scale is not in itself part of SI, it is common practice to specify temperatures in degrees Celsius when using SI units.

The force unit, called the newton (N), is defined from Newton’s second law as

\[ 1 \text{ N} = (1 \text{ kg})(1 \text{ m/s}^2) \]

Thus, a 1-N force acting on a 1-kg mass will give the mass an acceleration of 1 m/s^2. Standard gravity in SI is 9.807 m/s^2 (commonly approximated as 9.81 m/s^2) so that a 1-kg mass weighs 9.81 N under standard gravity. Note that weight and mass are different, both qualitatively and quantitatively! The unit of work in SI is the joule (J), which is the work done
Table 1.2
Prefixes for SI Units

<table>
<thead>
<tr>
<th>Factor by Which Unit Is Multiplied</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^{12}</td>
<td>tera</td>
<td>T</td>
</tr>
<tr>
<td>10^9</td>
<td>giga</td>
<td>G</td>
</tr>
<tr>
<td>10^6</td>
<td>mega</td>
<td>M</td>
</tr>
<tr>
<td>10^3</td>
<td>kilo</td>
<td>k</td>
</tr>
<tr>
<td>10^2</td>
<td>hecto</td>
<td>h</td>
</tr>
<tr>
<td>10</td>
<td>deka</td>
<td>da</td>
</tr>
<tr>
<td>10^{-1}</td>
<td>deci</td>
<td>d</td>
</tr>
<tr>
<td>10^{-2}</td>
<td>centi</td>
<td>c</td>
</tr>
<tr>
<td>10^{-3}</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>10^{-6}</td>
<td>micro</td>
<td>μ</td>
</tr>
<tr>
<td>10^{-9}</td>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>10^{-12}</td>
<td>pico</td>
<td>p</td>
</tr>
<tr>
<td>10^{-15}</td>
<td>femto</td>
<td>f</td>
</tr>
<tr>
<td>10^{-18}</td>
<td>atto</td>
<td>a</td>
</tr>
</tbody>
</table>

In mechanics it is very important to distinguish between weight and mass.

when the point of application of a 1-N force is displaced through a 1-m distance in the direction of a force. Thus,

\[ 1 \text{ J} = 1 \text{ N} \cdot \text{m} \]

The unit of power is the watt (W) defined as a joule per second. Thus,

\[ 1 \text{ W} = 1 \text{ J/s} = 1 \text{ N} \cdot \text{m/s} \]

Prefixes for forming multiples and fractions of SI units are given in Table 1.2. For example, the notation kN would be read as "kilonewtons" and stands for \(10^3\) N. Similarly, mm would be read as "millimeters" and stands for \(10^{-3}\) m. The centimeter is not an accepted unit of length in the SI system, so for most problems in fluid mechanics in which SI units are used, lengths will be expressed in millimeters or meters.

**English Engineering (EE) System.** In the EE system units for force and mass are defined independently; thus special care must be exercised when using this system in conjunction with Newton’s second law. The basic unit of mass is the pound mass (lbm), the unit of force is the pound (lb).\(^1\) The unit of length is the foot (ft), the unit of time is the second (s), and the absolute temperature scale is the degree Rankine (°R). To make the equation expressing Newton’s second law dimensionally homogeneous we write it as

\[ F = \frac{ma}{g_c} \tag{1.4} \]

where \(g_c\) is a constant of proportionality which allows us to define units for both force and mass. For the BG system only the force unit was prescribed and the mass unit defined in a

\(^1\)It is also common practice to use the notation, lbf, to indicate pound force.
consistent manner such that \( g_e = 1 \). Similarly, for SI the mass unit was prescribed and the force unit defined in a consistent manner such that \( g_e = 1 \). For the EE system, a 1-lb force is defined as that force which gives a 1 lbm a standard acceleration of gravity which is taken as 32.174 ft/s\(^2\). Thus, for Eq. 1.4 to be both numerically and dimensionally correct

\[
1 \text{ lb} = \frac{(1 \text{ lbm})(32.174 \text{ ft/s}^2)}{g_e}
\]

so that

\[
g_e = \frac{(1 \text{ lbm})(32.174 \text{ ft/s}^2)}{(1 \text{ lb})}
\]

With the EE system weight and mass are related through the equation

\[
W = \frac{mg}{g_e}
\]

where \( g \) is the local acceleration of gravity. Under conditions of standard gravity (\( g = g_e \)) the weight in pounds and the mass in pound mass are numerically equal. Also, since a 1-lb force gives a mass of 1 lbm an acceleration of 32.174 ft/s\(^2\), and a mass of 1 slug an acceleration of 1 ft/s\(^2\), it follows that

\[
1 \text{ slug} = 32.174 \text{ lbm}
\]

In this text we will primarily use the BG system and SI for units. The EE system is used very sparingly, and only in those instances where convention dictates its use. Approximately one-half the problems and examples are given in BG units and one-half in SI units. We cannot overemphasize the importance of paying close attention to units when solving problems. It is very easy to introduce huge errors into problem solutions through the use of incorrect units. Get in the habit of using a consistent system of units throughout a given solution. It really makes no difference which system you use as long as you are consistent; for example, don’t mix slugs and newtons. If problem data are specified in SI units, then use SI units throughout the solution. If the data are specified in BG units, then use BG units throughout the solution. Tables 1.3 and 1.4 provide conversion factors for some quantities that are commonly encountered in fluid mechanics. For convenient reference these tables are also reproduced on the inside of the back cover. Note that in these tables (and others) the numbers are expressed by using computer exponential notation. For example, the number 5.154 E + 2 is equivalent to 5.154 \( \times \) 10\(^2\) in scientific notation, and the number 2.832 E - 2 is equivalent to 2.832 \( \times \) 10\(^{-2}\). More extensive tables of conversion factors for a large variety of unit systems can be found in Appendix A.

| TABLE 1.3 |
| Conversion Factors from BG Units to SI Units |
| (See inside of back cover.) |

| TABLE 1.4 |
| Conversion Factors from SI Units to BG Units |
| (See inside of back cover.) |
A tank of water having a total mass of 36 kg rests on the floor of an elevator. Determine the force (in newtons) that the tank exerts on the floor when the elevator is accelerating upward at 7 ft/s².

**Solution**

A free-body diagram of the tank is shown in Fig. E1.2 where \( W \) is the weight of the tank and water, and \( F_f \) is the reaction of the floor on the tank. Application of Newton's second law of motion to this body gives

\[
\sum F = ma
\]

or

\[
F_f - W = ma \tag{1}
\]

where we have taken upward as the positive direction. Since \( W = mg \), Eq. 1 can be written as

\[
F_f = m(g + a) \tag{2}
\]

Before substituting any number into Eq. 2 we must decide on a system of units, and then be sure all of the data are expressed in these units. Since we want \( F_f \) in newtons we will use SI units so that

\[
F_f = 36 \text{ kg} \left[ 9.81 \text{ m/s}^2 + (7 \text{ ft/s}^2)(0.3048 \text{ m/ft}) \right] = 430 \text{ kg} \cdot \text{m/s}^2
\]

Since 1 N = 1 kg \cdot m/s² it follows that

\[
F_f = 430 \text{ N} \quad \text{(downward on floor)} \tag{Ans}
\]

The direction is downward since the force shown on the free-body diagram is the force of the floor on the tank so that the force the tank exerts on the floor is equal in magnitude but opposite in direction.

As you work through a large variety of problems in this text, you will find that units play an essential role in arriving at a numerical answer. Be careful! It is easy to mix units and cause large errors. If in the above example the elevator acceleration had been left as 7 ft/s² with \( m \) and \( g \) expressed in SI units, we would have calculated the force as 605 N and the answer would have been 41% too large!
1.3 Analysis of Fluid Behavior

The study of fluid mechanics involves the same fundamental laws you have encountered in physics and other mechanics courses. These laws include Newton's laws of motion, conservation of mass, and the first and second laws of thermodynamics. Thus, there are strong similarities between the general approach to fluid mechanics and to rigid-body and deformable-body solid mechanics. This is indeed helpful since many of the concepts and techniques of analysis used in fluid mechanics will be ones you have encountered before in other courses.

The broad subject of fluid mechanics can be generally subdivided into fluid statics, in which the fluid is at rest, and fluid dynamics, in which the fluid is moving. In the following chapters we will consider both of these areas in detail. Before we can proceed, however, it will be necessary to define and discuss certain fluid properties that are intimately related to fluid behavior. It is obvious that different fluids can have grossly different characteristics. For example, gases are light and compressible, whereas liquids are heavy (by comparison) and relatively incompressible. A syrup flows slowly from a container, but water flows rapidly when poured from the same container. To quantify these differences certain fluid properties are used. In the following several sections the properties that play an important role in the analysis of fluid behavior are considered.

1.4 Measures of Fluid Mass and Weight

1.4.1 Density

The density of a fluid, designated by the Greek symbol \( \rho \) (rho), is defined as its mass per unit volume. Density is typically used to characterize the mass of a fluid system. In the BG system \( \rho \) has units of slugs/\( ft^3 \) and in SI the units are kg/\( m^3 \).

The value of density can vary widely between different fluids, but for liquids, variations in pressure and temperature generally have only a small effect on the value of \( \rho \). The small change in the density of water with large variations in temperature is illustrated in Fig. 1.1. Tables 1.5 and 1.6 list values of density for several common liquids. The density of water at 60 °F is 1.94 slugs/\( ft^3 \) or 999 kg/\( m^3 \). The large difference between those two values illustrates the importance of paying attention to units! Unlike liquids, the density of a gas is strongly influenced by both pressure and temperature, and this difference will be discussed in the next section.

![Figure 1.1 Density of water as a function of temperature.](image-url)
The specific volume, $v$, is the volume per unit mass and is therefore the reciprocal of the density—that is,

$$v = \frac{1}{\rho} \quad (1.5)$$

This property is not commonly used in fluid mechanics but is used in thermodynamics.

### 1.4.2 Specific Weight

The specific weight of a fluid, designated by the Greek symbol $\gamma$ (gamma), is defined as its weight per unit volume. Thus, specific weight is related to density through the equation

$$\gamma = \rho g \quad (1.6)$$

where $g$ is the local acceleration of gravity. Just as density is used to characterize the mass of a fluid system, the specific weight is used to characterize the weight of the system. In the BG system, $\gamma$ has units of lb/ft$^3$ and in SI the units are N/m$^3$. Under conditions of standard gravity ($g = 32.174$ ft/s$^2 = 9.807$ m/s$^2$), water at 60 °F has a specific weight of 62.4 lb/ft$^3$ and 9.80 kN/m$^3$. Tables 1.5 and 1.6 list values of specific weight for several common liquids (based on standard gravity). More complete tables for water can be found in Appendix B (Tables B.1 and B.2).

### 1.4.3 Specific Gravity

The specific gravity of a fluid, designated as $SG$, is defined as the ratio of the density of the fluid to the density of water at some specified temperature. Usually the specified temperature is taken as 4 °C (39.2 °F), and at this temperature the density of water is 1.94 slugs/ft$^3$ or 1000 kg/m$^3$. In equation form, specific gravity is expressed as

$$SG = \frac{\rho}{\rho_{H_2O@4^\circ C}} \quad (1.7)$$

and since it is the ratio of densities, the value of $SG$ does not depend on the system of units used. For example, the specific gravity of mercury at 20 °C is 13.55 and the density of mercury can thus be readily calculated in either BG or SI units through the use of Eq. 1.7 as

$$\rho_{Hg} = (13.55)(1.94 \text{ slugs/ft}^3) = 26.3 \text{ slugs/ft}^3$$

or

$$\rho_{Hg} = (13.55)(1000 \text{ kg/m}^3) = 13.6 \times 10^3 \text{ kg/m}^3$$

It is clear that density, specific weight, and specific gravity are all interrelated, and from a knowledge of any one of the three the others can be calculated.
1.5 Ideal Gas Law

Gases are highly compressible in comparison to liquids, with changes in gas density directly related to changes in pressure and temperature through the equation

\[ p = \rho RT \]  

(1.8)

where \( p \) is the absolute pressure, \( \rho \) the density, \( T \) the absolute temperature,\(^2\) and \( R \) is a gas constant. Equation 1.8 is commonly termed the ideal or perfect gas law, or the equation of state for an ideal gas. It is known to closely approximate the behavior of real gases under normal conditions when the gases are not approaching liquefaction.

Pressure in a fluid at rest is defined as the normal force per unit area exerted on a plane surface (real or imaginary) immersed in a fluid and is created by the bombardment of the surface with the fluid molecules. From the definition, pressure has the dimension of \( FL^{-2} \), and in BG units is expressed as lb/ft\(^2\) (psf) or lb/in.\(^2\) (psi) and in SI units as N/m\(^2\). In SI, 1 N/m\(^2\) is defined as a pascal, abbreviated as Pa, and pressures are commonly specified in pascals. The pressure in the ideal gas law must be expressed as an absolute pressure, which means that it is measured relative to absolute zero pressure (a pressure that would only occur in a perfect vacuum). Standard sea-level atmospheric pressure (by international agreement) is 14.696 psi (abs) or 101.33 kPa (abs). For most calculations these pressures can be rounded to 14.7 psi and 101 kPa, respectively. In engineering it is common practice to measure pressure relative to the local atmospheric pressure, and when measured in this fashion it is called gage pressure. Thus, the absolute pressure can be obtained from the gage pressure by adding the value of the atmospheric pressure. For example, a pressure of 30 psi (gage) in a tire is equal to 44.7 psi (abs) at standard atmospheric pressure. Pressure is a particularly important fluid characteristic and it will be discussed more fully in the next chapter.

The gas constant, \( R \), which appears in Eq. 1.8, depends on the particular gas and is related to the molecular weight of the gas. Values of the gas constant for several common gases are listed in Tables 1.7 and 1.8. Also in these tables the gas density and specific weight are given for standard atmospheric pressure and gravity and for the temperature listed. More complete tables for air at standard atmospheric pressure can be found in Appendix B (Tables B.3 and B.4).

\[ \text{TABLE 1.7} \]
Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (BG Units)

(See inside of front cover.)

\[ \text{TABLE 1.8} \]
Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (SI Units)

(See inside of front cover.)

\(^2\)We will use \( T \) to represent temperature in thermodynamic relationships although \( T \) is also used to denote the basic dimension of time.
A compressed air tank has a volume of 0.84 ft³. When the tank is filled with air at a gage pressure of 50 psi, determine the density of the air and the weight of air in the tank. Assume the temperature is 70 °F and the atmospheric pressure is 14.7 psi (abs).

**Solution**

The air density can be obtained from the ideal gas law (Eq. 1.8) expressed as

\[ \rho = \frac{P}{RT} \]

so that

\[ \rho = \frac{(50 \text{ lb/in.}^2 + 14.7 \text{ lb/in.}^2)(144 \text{ in.}^2/\text{ft}^2)}{(1716 \text{ ft-lbslug.}^o\text{R})(70 + 460)^o\text{R}} = 0.0102 \text{ slugs/ft}^3 \quad \text{(Ans)} \]

Note that both the pressure and temperature were changed to absolute values.

The weight, \( \mathcal{W} \), of the air is equal to

\[ \mathcal{W} = \rho g \times \text{(volume)} \]

\[ = (0.0102 \text{ slugs/ft}^3)(32.2 \text{ ft/s}^2)(0.84 \text{ ft}^3) \]

so that since 1 lb = 1 slug·ft/s²

\[ \mathcal{W} = 0.276 \text{ lb} \quad \text{(Ans)} \]

1.6 Viscosity

The properties of density and specific weight are measures of the "heaviness" of a fluid. It is clear, however, that these properties are not sufficient to uniquely characterize how fluids behave since two fluids (such as water and oil) can have approximately the same value of density but behave quite differently when flowing. There is apparently some additional property that is needed to describe the "fluidity" of the fluid.

To determine this additional property, consider a hypothetical experiment in which a material is placed between two very wide parallel plates as shown in Fig. 1.2a. The bottom plate is rigidly fixed, but the upper plate is free to move. If a solid, such as steel, were placed between the two plates and loaded with the force \( P \) as shown, the top plate would be displaced through some small distance, \( \delta \alpha \) (assuming the solid was mechanically attached to the plates). The vertical line \( AB \) would be rotated through the small angle, \( \delta \beta \), to the new position \( AB' \). We note that to resist the applied force, \( P \), a shearing stress, \( \tau \), would be developed at the plate-material interface, and for equilibrium to occur \( P = \tau A \) where \( A \) is the effective upper
plate area (Fig. 1.2b). It is well known that for elastic solids, such as steel, the small angular displacement, $\delta \beta$ (called the shearing strain), is proportional to the shearing stress, $\tau$, that is developed in the material.

What happens if the solid is replaced with a fluid such as water? We would immediately notice a major difference. When the force $P$ is applied to the upper plate, it will move continuously with a velocity, $U$ (after the initial transient motion has died out) as illustrated in Fig. 1.3. This behavior is consistent with the definition of a fluid—that is, if a shearing stress is applied to a fluid it will deform continuously. A closer inspection of the fluid motion between the two plates would reveal that the fluid in contact with the upper plate moves with the plate velocity, $U$, and the fluid in contact with the bottom fixed plate has a zero velocity. The fluid between the two plates moves with velocity $u = u(y)$ that would be found to vary linearly, $u = Uy/b$, as illustrated in Fig. 1.3. Thus, a velocity gradient, $du/\delta y$, is developed in the fluid between the plates. In this particular case the velocity gradient is a constant since $du/\delta y = U/b$, but in more complex flow situations this would not be true. The experimental observation that the fluid "sticks" to the solid boundaries is a very important one in fluid mechanics and is usually referred to as the no-slip condition. All fluids, both liquids and gases, satisfy this condition.

In a small time increment, $\delta t$, an imaginary vertical line $AB$ in the fluid would rotate through an angle, $\delta \beta$, so that

$$\tan \delta \beta \approx \delta \beta = \frac{\delta a}{b}$$

Since $\delta a = U \delta t$ it follows that

$$\delta \beta = \frac{U \delta t}{b}$$

We note that in this case, $\delta \beta$ is a function not only of the force $P$ (which governs $U$) but also of time. Thus, it is not reasonable to attempt to relate the shearing stress, $\tau$, to $\delta \beta$ as is done for solids. Rather, we consider the rate at which $\delta \beta$ is changing and define the rate of shearing strain, $\dot{\gamma}$, as

$$\dot{\gamma} = \lim_{\delta t \to 0} \frac{\delta \beta}{\delta t}$$

which in this instance is equal to

$$\dot{\gamma} = \frac{U}{b} = \frac{du}{dy}$$

A continuation of this experiment would reveal that as the shearing stress, $\tau$, is increased by increasing $P$ (recall that $\tau = P/A$), the rate of shearing strain is increased in direct proportion—that is

![Figure 1.3](image-url)

**Figure 1.3** Behavior of a fluid placed between two parallel plates.
\[ \tau \propto \dot{\gamma} \]

or

\[ \tau \propto \frac{du}{dy} \]

This result indicates that for common fluids such as water, oil, gasoline, and air the shearing stress and rate of shearing strain (velocity gradient) can be related with a relationship of the form

\[ \tau = \mu \frac{du}{dy} \tag{1.9} \]

where the constant of proportionality is designated by the Greek symbol \( \mu \) (mu) and is called the absolute viscosity, dynamic viscosity, or simply the viscosity of the fluid. In accordance with Eq. 1.9, plots of \( \tau \) versus \( \frac{du}{dy} \) should be linear with the slope equal to the viscosity as illustrated in Fig. 1.4. The actual value of the viscosity depends on the particular fluid, and for a particular fluid the viscosity is also highly dependent on temperature as illustrated in Fig. 1.4 with the two curves for water. Fluids for which the shearing stress is linearly related to the rate of shearing strain (also referred to as rate of angular deformation) are designated as Newtonian fluids. Fortunately most common fluids, both liquids and gases, are Newtonian.

A more general formulation of Eq. 1.9 which applies to more complex flows of Newtonian fluids is given in Section 6.8.1.

Fluids for which the shearing stress is not linearly related to the rate of shearing strain are designated as non-Newtonian fluids. Although there is a variety of types of non-Newtonian fluids, the simplest and most common are shown in Fig. 1.5. The slope of the shearing stress vs rate of shearing strain graph is denoted as the apparent viscosity, \( \mu_{app} \). For Newtonian fluids the apparent viscosity is the same as the viscosity and is independent of shear rate.

For shear thinning fluids the apparent viscosity decreases with increasing shear rate—the harder the fluid is sheared, the less viscous it becomes. Many colloidal suspensions and polymer solutions are shear thinning. For example, latex paint does not drip from the brush because the shear rate is small and the apparent viscosity is large. However, it flows smoothly.
onto the wall because the thin layer of paint between the wall and the brush causes a large shear rate (large $du/dy$) and a small apparent viscosity.

For shear thickening fluids the apparent viscosity increases with increasing shear rate—the harder the fluid is sheared, the more viscous it becomes. Common examples of this type of fluid include water-corn starch mixture and water-sand mixture ("quicksand"). Thus, the difficulty in removing an object from quicksand increases dramatically as the speed of removal increases.

The other type of behavior indicated in Fig. 1.5 is that of a Bingham plastic, which is neither a fluid nor a solid. Such material can withstand a finite shear stress without motion (therefore, it is not a fluid), but once the yield stress is exceeded it flows like a fluid (hence, it is not a solid). Toothpaste and mayonnaise are common examples of Bingham plastic materials.

From Eq. 1.9 it can be readily deduced that the dimensions of viscosity are $F T L^{-2}$. Thus, in BG units viscosity is given as lb·s/ft² and in SI units as N·s/m². Values of viscosity for several common liquids and gases are listed in Tables 1.5 through 1.8. A quick glance at these tables reveals the wide variation in viscosity among fluids. Viscosity is only mildly dependent on pressure and the effect of pressure is usually neglected. However, as previously mentioned, and as illustrated in Fig. 1.6, viscosity is very sensitive to temperature. For example, as the temperature of water changes from 60 to 100 °F the density decreases by less than 1% but the viscosity decreases by about 40%. It is thus clear that particular attention must be given to temperature when determining viscosity.

Figure 1.6 shows in more detail how the viscosity varies from fluid to fluid and how for a given fluid it varies with temperature. It is to be noted from this figure that the viscosity of liquids decreases with an increase in temperature, whereas for gases an increase in temperature causes an increase in viscosity. This difference in the effect of temperature on the viscosity of liquids and gases can again be traced back to the difference in molecular structure. The liquid molecules are closely spaced, with strong cohesive forces between molecules, and the resistance to relative motion between adjacent layers of fluid is related to these intermolecular forces. As the temperature increases, these cohesive forces are reduced with a corresponding reduction in resistance to motion. Since viscosity is an index of this resistance, it follows that the viscosity is reduced by an increase in temperature. In gases, however, the molecules are widely spaced and intermolecular forces negligible. In this case resistance to relative motion arises due to the exchange of momentum of gas molecules between adjacent layers. As molecules are transported by random motion from a region of low bulk velocity
to mix with molecules in a region of higher bulk velocity (and vice versa), there is an effective momentum exchange which resists the relative motion between the layers. As the temperature of the gas increases, the random molecular activity increases with a corresponding increase in viscosity.

The effect of temperature on viscosity can be closely approximated using two empirical formulas. For gases the Sutherland equation can be expressed as

$$\mu = \frac{CT^{3/2}}{T + S}$$  \hspace{1cm} (1.10)

where $C$ and $S$ are empirical constants, and $T$ is absolute temperature. Thus, if the viscosity is known at two temperatures, $C$ and $S$ can be determined. Or, if more than two viscosities are known, the data can be correlated with Eq. 1.10 by using some type of curve-fitting scheme.

For liquids an empirical equation that has been used is

$$\mu = De^{B/T}$$  \hspace{1cm} (1.11)

where $D$ and $B$ are constants and $T$ is absolute temperature. This equation is often referred to as Andrade's equation. As was the case for gases, the viscosity must be known at least for two temperatures so the two constants can be determined. A more detailed discussion of the effect of temperature on fluids can be found in Ref. 1.
Quite often viscosity appears in fluid flow problems combined with the density in the form

\[ \nu = \frac{\mu}{\rho} \]

This ratio is called the **kinematic viscosity** and is denoted with the Greek symbol \( \nu \) (nu). The dimensions of kinematic viscosity are \( L^2/T \), and the BG units are \( ft^2/s \) and SI units are \( m^2/s \). Values of kinematic viscosity for some common liquids and gases are given in Tables 1.5 through 1.8. More extensive tables giving both the dynamic and kinematic viscosities for water and air can be found in Appendix B (Tables B.1 through B.4), and graphs showing the variation in both dynamic and kinematic viscosity with temperature for a variety of fluids are also provided in Appendix B (Figs. B.1 and B.2).

Although in this text we are primarily using BG and SI units, dynamic viscosity is often expressed in the metric CGS (centimeter-gram-second) system with units of dyne-s/cm². This combination is called a **poise**, abbreviated P. In the CGS system, kinematic viscosity has units of cm²/s, and this combination is called a **stoke**, abbreviated St.

**Example 1.4**

A dimensionless combination of variables that is important in the study of viscous flow through pipes is called the **Reynolds number**, Re, defined as \( \rho V D/\mu \) where \( \rho \) is the fluid density, \( V \) the mean fluid velocity, \( D \) the pipe diameter, and \( \mu \) the fluid viscosity. A Newtonian fluid having a viscosity of 0.38 N-s/m² and a specific gravity of 0.91 flows through a 25-mm-diameter pipe with a velocity of 2.6 m/s. Determine the value of the Reynolds number using (a) SI units, and (b) BG units.

**Solution**

(a) The fluid density is calculated from the specific gravity as

\[ \rho = SG \rho _{H_2O at 4^\circ C} = 0.91 \times 1000 \text{ kg/m}^3 = 910 \text{ kg/m}^3 \]

and from the definition of the Reynolds number

\[ \text{Re} = \frac{\rho V D}{\mu} = \frac{(910 \text{ kg/m}^3)(2.6 \text{ m/s})(25 \text{ mm})(10^{-3} \text{ m/mm})}{0.38 \text{ N-s/m}^2} \]

\[ = 156 \text{ (kg-m/s²)/N} \]

However, since 1 N = 1 kg-m/s² it follows that the Reynolds number is unitless—that is,

\[ \text{Re} = 156 \quad (\text{Ans}) \]

The value of any dimensionless quantity does not depend on the system of units used if all variables that make up the quantity are expressed in a consistent set of units. To check this we will calculate the Reynolds number using BG units.

(b) We first convert all the SI values of the variables appearing in the Reynolds number to BG values by using the conversion factors from Table 1.4. Thus,

\[ \rho = (910 \text{ kg/m}^3)(1.940 \times 10^{-3}) = 1.77 \text{ slugs/ft}^3 \]

\[ V = (2.6 \text{ m/s})(3.281) = 8.53 \text{ ft/s} \]

\[ D = (0.025 \text{ m})(3.281) = 8.20 \times 10^{-2} \text{ ft} \]

\[ \mu = (0.38 \text{ N-s/m}^2)(2.089 \times 10^{-2}) = 7.94 \times 10^{-3} \text{ lb-s/ft}^2 \]
and the value of the Reynolds number is
\[
Re = \frac{(1.77 \text{ slugs}/\text{ft}^3)(8.53 \text{ ft/s})(8.20 \times 10^{-2} \text{ ft})}{7.94 \times 10^{-5} \text{ lb-s/ft}^2} = 156 \text{ (slug-ft/s}^2) / \text{lb} = 156
\]

since 1 lb = 1 slug-ft/s^2. The values from part (a) and part (b) are the same, as expected. Dimensionless quantities play an important role in fluid mechanics and the significance of the Reynolds number as well as other important dimensionless combinations will be discussed in detail in Chapter 7. It should be noted that in the Reynolds number it is actually the ratio \( \mu/p \) that is important, and this is the property that we have defined as the kinematic viscosity.

**Example 1.5**

The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates (see Fig. E1.5) is given by the equation
\[
u = \frac{3V}{2} \left[ 1 - \left( \frac{y}{h} \right)^2 \right]
\]
where \( V \) is the mean velocity. The fluid has a viscosity of 0.04 lb-s/ft^2. When \( V = 2 \text{ ft/s} \) and \( h = 0.2 \text{ in.} \) determine: (a) the shearing stress acting on the bottom wall, and (b) the shearing stress acting on a plane parallel to the walls and passing through the centerline (midplane).

**Solution**

For this type of parallel flow the shearing stress is obtained from Eq. 1.9,
\[
\tau = \mu \frac{du}{dy}
\]

Thus, if the velocity distribution, \( u = u(y) \) is known, the shearing stress can be determined at all points by evaluating the velocity gradient, \( du/dy \). For the distribution given
\[
\frac{du}{dy} = -\frac{3Vy}{h^2}
\]

(a) Along the bottom wall \( y = -h \) so that (from Eq. 2)
\[
\frac{du}{dy} = \frac{3V}{h}
\]
and therefore the shearing stress is

\[
\tau_{\text{bottom wall}} = \mu \frac{3V}{h} = \frac{(0.04 \text{ lb/s/ft}^2)(3\text{ ft/s})}{(0.2 \text{ in.})(1 \text{ ft/12 in.})} = 14.4 \text{ lb/ft}^2 \text{ (in direction of flow)} \quad \text{(Ans)}
\]

This stress creates a drag on the wall. Since the velocity distribution is symmetrical, the shearing stress along the upper wall would have the same magnitude and direction.

(b) Along the midplane where \( y = 0 \) it follows from Eq. 2 that

\[
\frac{du}{dy} = 0
\]

and thus the shearing stress is

\[
\tau_{\text{midplane}} = 0 \quad \text{(Ans)}
\]

From Eq. 2 we see that the velocity gradient (and therefore the shearing stress) varies linearly with \( y \) and in this particular example varies from 0 at the center of the channel to 14.4 lb/ft\(^2\) at the walls. For the more general case the actual variation will, of course, depend on the nature of the velocity distribution.

1.7 Compressibility of Fluids

1.7.1 Bulk Modulus

An important question to answer when considering the behavior of a particular fluid is how easily can the volume (and thus the density) of a given mass of the fluid be changed when there is a change in pressure? That is, how compressible is the fluid? A property that is commonly used to characterize compressibility is the bulk modulus, \( E_v \), defined as

\[
E_v = -\frac{dp}{dV/V} \quad \text{(1.12)}
\]

where \( dp \) is the differential change in pressure needed to create a differential change in volume, \( dV \), of a volume \( V \). The negative sign is included since an increase in pressure will cause a decrease in volume. Since a decrease in volume of a given mass, \( m = \rho V \), will result in an increase in density, Eq. 1.12 can also be expressed as

\[
E_v = \frac{dp}{d\rho/\rho} \quad \text{(1.13)}
\]

The bulk modulus (also referred to as the bulk modulus of elasticity) has dimensions of pressure, \( FL^{-2} \). In BG units values for \( E_v \) are usually given as lb/in.\(^2\) (psi) and in SI units as N/m\(^2\) (Pa). Large values for the bulk modulus indicate that the fluid is relatively incompressible—that is, it takes a large pressure change to create a small change in volume. As expected, values of \( E_v \) for common liquids are large (see Tables 1.5 and 1.6). For example, at atmospheric pressure and a temperature of 60 °F it would require a pressure of 3120 psi to compress a unit volume of water 1%. This result is representative of the compressibility of liquids. Since such large pressures are required to effect a change in volume, we conclude that liquids can be considered as incompressible for most practical engineering applications.
As liquids are compressed the bulk modulus increases, but the bulk modulus near atmospheric pressure is usually the one of interest. The use of bulk modulus as a property describing compressibility is most prevalent when dealing with liquids, although the bulk modulus can also be determined for gases.

### 1.7.2 Compression and Expansion of Gases

When gases are compressed (or expanded) the relationship between pressure and density depends on the nature of the process. If the compression or expansion takes place under constant temperature conditions (*isothermal process*), then from Eq. 1.8

$$\frac{P}{\rho} = \text{constant} \quad (1.14)$$

If the compression or expansion is frictionless and no heat is exchanged with the surroundings (*isentropic process*), then

$$\frac{P}{\rho^k} = \text{constant} \quad (1.15)$$

where $k$ is the ratio of the specific heat at constant pressure, $c_p$, to the specific heat at constant volume, $c_v$ (i.e., $k = c_p/c_v$). The two specific heats are related to the gas constant, $R$, through the equation $R = c_p - c_v$. As was the case for the ideal gas law, the pressure in both Eqs. 1.14 and 1.15 must be expressed as an absolute pressure. Values of $k$ for some common gases are given in Tables 1.7 and 1.8, and for air over a range of temperatures, in Appendix B (Tables B.3 and B.4).

With explicit equations relating pressure and density the bulk modulus for gases can be determined by obtaining the derivative $dp/d\rho$ from Eq. 1.14 or 1.15 and substituting the results into Eq. 1.13. It follows that for an isothermal process

$$E_v = p \quad (1.16)$$

and for an isentropic process

$$E_v = kp \quad (1.17)$$

Note that in both cases the bulk modulus varies directly with pressure. For air under standard atmospheric conditions with $p = 14.7$ psi (abs) and $k = 1.40$, the isentropic bulk modulus is 20.6 psi. A comparison of this figure with that for water under the same conditions ($E_v = 312,000$ psi) shows that air is approximately 15,000 times as compressible as water. It is thus clear that in dealing with gases greater attention will need to be given to the effect of compressibility on fluid behavior. However, as will be discussed further in later sections, gases can often be treated as incompressible fluids if the changes in pressure are small.

#### Example 1.6

A cubic foot of helium at an absolute pressure of 14.7 psi is compressed isentropically to $\frac{1}{2}$ ft$^3$. What is the final pressure?

**Solution**

For an isentropic compression

$$\frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$$
where the subscripts $i$ and $f$ refer to initial and final states, respectively. Since we are interested in the final pressure, $p_f$, it follows that

$$p_f = \left( \frac{p_f}{p_i} \right)^k p_i$$

As the volume is reduced by one half, the density must double, since the mass of the gas remains constant. Thus,

$$p_f = (2)^{1.66}(14.7 \text{ psi}) = 46.5 \text{ psi (abs)} \quad \text{(Ans)}$$

### 1.7.3 Speed of Sound

Another important consequence of the compressibility of fluids is that disturbances introduced at some point in the fluid propagate at a finite velocity. For example, if a fluid is flowing in a pipe and a valve at the outlet is suddenly closed (thereby creating a localized disturbance), the effect of the valve closure is not felt instantaneously upstream. It takes a finite time for the increased pressure created by the valve closure to propagate to an upstream location. Similarly, a loud speaker diaphragm causes a localized disturbance as it vibrates, and the small change in pressure created by the motion of the diaphragm is propagated through the air with a finite velocity. The velocity at which these small disturbances propagate is called the *acoustic velocity* or the *speed of sound*, $c$. It will be shown in Chapter 11 that the speed of sound is related to changes in pressure and density of the fluid medium through the equation

$$c = \sqrt{\frac{dp}{d\rho}} \quad \text{(1.18)}$$

or in terms of the bulk modulus defined by Eq. 1.13

$$c = \sqrt{\frac{E_v}{\rho}} \quad \text{(1.19)}$$

Since the disturbance is small, there is negligible heat transfer and the process is assumed to be isentropic. Thus, the pressure-density relationship used in Eq. 1.18 is that for an isentropic process.

For gases undergoing an isentropic process, $E_v = kp$ (Eq. 1.17) so that

$$c = \sqrt{\frac{kp}{\rho}}$$

and making use of the ideal gas law, it follows that

$$c = \sqrt{\frac{kRT}{\rho}} \quad \text{(1.20)}$$

Thus, for ideal gases the speed of sound is proportional to the square root of the absolute temperature. For example, for air at 60 °F with $k = 1.40$ and $R = 1716 \text{ ft-lb/slug} \cdot \text{°R}$ it follows that $c = 1117 \text{ ft/s}$. The speed of sound in air at various temperatures can be found in Appendix B (Tables B.3 and B.4). Equation 1.19 is also valid for liquids, and values of $E_v$ can be used to determine the speed of sound in liquids. For water at 20 °C, $E_v = 2.19 \text{ GN/m}^2$ and $\rho = 998.2 \text{ kg/m}^3$ so that $c = 1481 \text{ m/s}$ or 4860 ft/s. Note that the speed of sound in water is much higher than in air. If a fluid were truly incompressible ($E_v = \infty$) the speed of sound would be infinite. The speed of sound in water for various temperatures can be found in Appendix B (Tables B.1 and B.2).
A jet aircraft flies at a speed of 550 mph at an altitude of 35,000 ft, where the temperature is 
−66 °F. Determine the ratio of the speed of the aircraft, \( V \), to that of the speed of sound, \( c \), 
at the specified altitude. Assume \( k = 1.40 \).

**Solution**

From Eq. 1.20 the speed of sound can be calculated as

\[
c = \sqrt{kRT} = \sqrt{(1.40)(1716 \text{ ft-lb/slug·°R})(−66 + 460) \text{ °R}}
\]

\[
= 973 \text{ ft/s}
\]

Since the air speed is

\[
V = \frac{(550 \text{ mi/hr})(5280 \text{ ft/mi})}{(3600 \text{ s/hr})} = 807 \text{ ft/s}
\]

the ratio is

\[
\frac{V}{c} = \frac{807 \text{ ft/s}}{973 \text{ ft/s}} = 0.829
\]

This ratio is called the *Mach number*, \( Ma \). If \( Ma < 1.0 \) the aircraft is flying at subsonic speeds, 
whereas for \( Ma > 1.0 \) it is flying at supersonic speeds. The Mach number is an important 
dimensionless parameter used in the study of the flow of gases at high speeds and will be 
further discussed in Chapters 7, 9, and 11.

### 1.8 Vapor Pressure

It is a common observation that liquids such as water and gasoline will evaporate if they are simply placed in a container open to the atmosphere. Evaporation takes place because some liquid molecules at the surface have sufficient momentum to overcome the intermolecular cohesive forces and escape into the atmosphere. If the container is closed with a small air space left above the surface, and this space evacuated to form a vacuum, a pressure will develop in the space as a result of the vapor that is formed by the escaping molecules. When an equilibrium condition is reached so that the number of molecules leaving the surface is equal to the number entering, the vapor is said to be saturated and the pressure that the vapor exerts on the liquid surface is termed the vapor pressure.

Since the development of a vapor pressure is closely associated with molecular activity, the value of vapor pressure for a particular liquid depends on temperature. Values of vapor pressure for water at various temperatures can be found in Appendix B (Tables B.1 and B.2), and the values of vapor pressure for several common liquids at room temperatures are given in Tables 1.5 and 1.6.

*Boiling*, which is the formation of vapor bubbles within a fluid mass, is initiated when the absolute pressure in the fluid reaches the vapor pressure. As commonly observed in the kitchen, water at standard atmospheric pressure will boil when the temperature reaches 212 °F (100 °C)—that is, the vapor pressure of water at 212 °F is 14.7 psi (abs). However, if we attempt to boil water at a higher elevation, say 10,000 ft above sea level, where the atmospheric pressure is 10.1 psi (abs), we find that boiling will start when the temperature is about
193 °F. At this temperature the vapor pressure of water is 10.1 psi (abs). Thus, boiling can be induced at a given pressure acting on the fluid by raising the temperature, or at a given fluid temperature by lowering the pressure.

An important reason for our interest in vapor pressure and boiling lies in the common observation that in flowing fluids it is possible to develop very low pressure due to the fluid motion, and if the pressure is lowered to the vapor pressure, boiling will occur. For example, this phenomenon may occur in flow through the irregular, narrowed passages of a valve or pump. When vapor bubbles are formed in a flowing fluid they are swept along into regions of higher pressure, these suddenly collapse with sufficient intensity to actually cause structural damage. The formation and subsequent collapse of vapor bubbles in a flowing fluid, called cavitation, is an important fluid flow phenomenon to be given further attention in Chapters 3 and 7.

1.9 Surface Tension

At the interface between a liquid and a gas, or between two immiscible liquids, forces develop in the liquid surface which cause the surface to behave as if it were a "skin" or "membrane" stretched over the fluid mass. Although such a skin is not actually present, this conceptual analogy allows us to explain several commonly observed phenomena. For example, a steel needle will float on water if placed gently on the surface because the tension developed in the hypothetical skin supports the needle. Small droplets of mercury will form into spheres when placed on a smooth surface because the cohesive forces in the surface tend to hold all the molecules together in a compact shape. Similarly, discrete water droplets will form when placed on a newly waxed surface.

These various types of surface phenomena are due to the unbalanced cohesive forces acting on the liquid molecules at the fluid surface. Molecules in the interior of the fluid mass are surrounded by molecules that are attracted to each other equally. However, molecules along the surface are subjected to a net force toward the interior. The apparent physical consequence of this unbalanced force along the surface is to create the hypothetical skin or membrane. A tensile force may be considered to be acting in the plane of the surface along any line in the surface. The intensity of the molecular attraction per unit length along any line in the surface is called the surface tension and is designated by the Greek symbol \( \sigma \) (sigma). For a given liquid the surface tension depends on temperature as well as the other fluid it is in contact with at the interface. The dimensions of surface tension are \( FL^{-1} \) with BG units of lb/ft and SI units of N/m. Values of surface tension for some common liquids (in contact with air) are given in Tables 1.5 and 1.6 and in Appendix B (Tables B.1 and B.2) for water at various temperatures. The value of the surface tension decreases as the temperature increases.

The pressure inside a drop of fluid can be calculated using the free-body diagram in Fig. 1.7. If the spherical drop is cut in half (as shown) the force developed around the edge

\[ F = 2 \pi R \sigma \]

**FIGURE 1.7** Forces acting on one-half of a liquid drop.
due to surface tension is $2\pi R\sigma$. This force must be balanced by the pressure difference, $\Delta p$, between the internal pressure, $p_i$, and the external pressure, $p_e$, acting over the circular area, $\pi R^2$. Thus,

$$2\pi R\sigma = \Delta p \pi R^2$$

or

$$\Delta p = p_i - p_e = \frac{2\sigma}{R}$$  \hspace{1cm} (1.21)

It is apparent from this result that the pressure inside the drop is greater than the pressure surrounding the drop. (Would the pressure on the inside of a bubble of water be the same as that on the inside of a drop of water of the same diameter and at the same temperature?)

Among common phenomena associated with surface tension is the rise (or fall) of a liquid in a capillary tube. If a small open tube is inserted into water, the water level in the tube will rise above the water level outside the tube as is illustrated in Fig. 1.8a. In this situation we have a liquid–gas–solid interface. For the case illustrated there is an attraction (adhesion) between the wall of the tube and liquid molecules which is strong enough to overcome the mutual attraction (cohesion) of the molecules and pull them up the wall. Hence, the liquid is said to wet the solid surface.

The height, $h$, is governed by the value of the surface tension, $\sigma$, the tube radius, $R$, the specific weight of the liquid, $\gamma$, and the angle of contact, $\theta$, between the fluid and tube. From the free-body diagram of Fig. 1.8b we see that the vertical force due to the surface tension is equal to $2\pi R\sigma \cos \theta$ and the weight is $\gamma R^2 h$ and these two forces must balance for equilibrium. Thus,

$$\gamma R^2 h = 2\pi R\sigma \cos \theta$$

so that the height is given by the relationship

$$h = \frac{2\sigma \cos \theta}{\gamma R}$$  \hspace{1cm} (1.22)

The angle of contact is a function of both the liquid and the surface. For water in contact with clean glass $\theta \approx 0^\circ$. It is clear from Eq. 1.22 that the height is inversely proportional to the tube radius, and therefore the rise of a liquid in a tube as a result of capillary action becomes increasingly pronounced as the tube radius is decreased.

**Figure 1.8** Effect of capillary action in small tubes. (a) Rise of column for a liquid that wets the tube. (b) Free-body diagram for calculating column height. (c) Depression of column for a nonwetting liquid.
Pressures are sometimes determined by measuring the height of a column of liquid in a vertical tube. What diameter of clean glass tubing is required so that the rise of water at 20 °C in a tube due to capillary action (as opposed to pressure in the tube) is less than 1.0 mm?

**Solution**

From Eq. 1.22

\[ h = \frac{2\sigma \cos \theta}{\gamma R} \]

so that

\[ R = \frac{2\sigma \cos \theta}{\gamma h} \]

For water at 20 °C (from Table B.2), \( \sigma = 0.0728 \text{ N/m} \) and \( \gamma = 9.789 \text{ kN/m}^2 \). Since \( \theta \approx 0^\circ \) it follows that for \( h = 1.0 \text{ mm} \),

\[ R = \frac{2(0.0728 \text{ N/m})(1)}{(9.789 \times 10^3 \text{ N/m}^2)(1.0 \text{ mm})(10^{-3} \text{ m/mm})} = 0.0149 \text{ m} \]

and the minimum required tube diameter, \( D \), is

\[ D = 2R = 0.0298 \text{ m} = 29.8 \text{ mm} \] (Ans)

If adhesion of molecules to the solid surface is weak compared to the cohesion between molecules, the liquid will not wet the surface and the level in a tube placed in a nonwetting liquid will actually be depressed as shown in Fig. 1.8c. Mercury is a good example of a nonwetting liquid when it is in contact with a glass tube. For nonwetting liquids the angle of contact is greater than 90°, and for mercury in contact with clean glass \( \theta \approx 130° \).

Surface tension effects play a role in many fluid mechanics problems including the movement of liquids through soil and other porous media, flow of thin films, formation of drops and bubbles, and the breakup of liquid jets. Surface phenomena associated with liquid-gas, liquid-liquid, liquid-gas-solid interfaces are exceedingly complex, and a more detailed and rigorous discussion of them is beyond the scope of this text. Fortunately, in many fluid mechanics problems, surface phenomena, as characterized by surface tension, are not important, since inertial, gravitational, and viscous forces are much more dominant.

### 1.10 A Brief Look Back in History

Before proceeding with our study of fluid mechanics, we should pause for a moment to consider the history of this important engineering science. As is true of all basic scientific and engineering disciplines, their actual beginnings are only faintly visible through the haze of early antiquity. But, we know that interest in fluid behavior dates back to the ancient civilizations. Through necessity there was a practical concern about the manner in which spears and arrows could be propelled through the air, in the development of water supply and irrigation systems, and in the design of boats and ships. These developments were of course based on trial and error procedures without any knowledge of mathematics or mechanics.
However, it was the accumulation of such empirical knowledge that formed the basis for further development during the emergence of the ancient Greek civilization and the subsequent rise of the Roman Empire. Some of the earliest writings that pertain to modern fluid mechanics are those of Archimedes (287–212 B.C.), a Greek mathematician and inventor who first expressed the principles of hydrostatics and flotation. Elaborate water supply systems were built by the Romans during the period from the fourth century B.C. through the early Christian period, and Sextus Julius Frontinus (A.D. 40–103), a Roman engineer, described these systems in detail. However, for the next 1000 years during the Middle Ages (also referred to as the Dark Ages), there appears to have been little added to further understanding of fluid behavior.

Beginning with the Renaissance period (about the fifteenth century) a rather continuous series of contributions began that forms the basis of what we consider to be the science of fluid mechanics. Leonardo da Vinci (1452–1519) described through sketches and writings many different types of flow phenomena. The work of Galileo Galilei (1564–1642) marked the beginning of experimental mechanics. Following the early Renaissance period and during the seventeenth and eighteenth centuries, numerous significant contributions were made. These include theoretical and mathematical advances associated with the famous names of Newton, Bernoulli, Euler, and d'Alembert. Experimental aspects of fluid mechanics were also advanced during this period, but unfortunately the two different approaches, theoretical and experimental, developed along separate paths. Hydrodynamics was the term associated with the theoretical or mathematical study of idealized, frictionless fluid behavior, with the term hydraulics being used to describe the applied or experimental aspects of real fluid behavior, particularly the behavior of water. Further contributions and refinements were made to both theoretical hydrodynamics and experimental hydraulics during the nineteenth century, with the general differential equations describing fluid motions that are used in modern fluid mechanics being developed in this period. Experimental hydraulics became more of a science, and many of the results of experiments performed during the nineteenth century are still used today.

At the beginning of the twentieth century both the fields of theoretical hydrodynamics and experimental hydraulics were highly developed, and attempts were being made to unify the two. In 1904 a classic paper was presented by a German professor, Ludwig Prandtl (1857–1953), who introduced the concept of a “fluid boundary layer,” which laid the foundation for the unification of the theoretical and experimental aspects of fluid mechanics. Prandtl's idea was that for flow next to a solid boundary a thin fluid layer (boundary layer) develops in which friction is very important, but outside this layer the fluid behaves very much like a frictionless fluid. This relatively simple concept provided the necessary impetus for the resolution of the conflict between the hydrodynamists and the hydraulicists. Prandtl is generally accepted as the founder of modern fluid mechanics.

Also, during the first decade of the twentieth century, powered flight was first successfully demonstrated with the subsequent vastly increased interest in aerodynamics. Because the design of aircraft required a degree of understanding of fluid flow and an ability to make accurate predictions of the effect of air flow on bodies, the field of aerodynamics provided a great stimulus for the many rapid developments in fluid mechanics that have taken place during the twentieth century.

As we proceed with our study of the fundamentals of fluid mechanics, we will continue to note the contributions of many of the pioneers in the field. Table 1.9 provides a chronological listing of some of these contributors and reveals the long journey that makes up the history of fluid mechanics. This list is certainly not comprehensive with regard to all of the past contributors, but includes those who are mentioned in this text. As mention is made in succeeding chapters of the various individuals listed in Table 1.9, a quick glance at this table will reveal where they fit into the historical chain.
| **TABLE 1.9**  
<table>
<thead>
<tr>
<th>Chronological Listing of Some Contributors to the Science of Fluid Mechanics Noted in the Text*</th>
</tr>
</thead>
</table>
| **ARCHIMEDES** (287–212 B.C.)  
Established elementary principles of buoyancy and flotation. |
| **SEXTUS JULIUS FRONTINUS** (A.D. 40–103)  
Wrote treatise on Roman methods of water distribution. |
| **LEONARDO da VINCI** (1452–1519)  
Expressed elementary principle of continuity; observed and sketched many basic flow phenomena; suggested designs for hydraulic machinery. |
| **GALILEO GALILEI** (1564–1642)  
Indirectly stimulated experimental hydraulics; revised Aristotelian concept of vacuum. |
| **EVANGELISTA TORRICELLI** (1608–1647)  
Related barometric height to weight of atmosphere, and form of liquid jet to trajectory of free fall. |
| **BLAISE PASCAL** (1623–1662)  
Finally clarified principles of barometer, hydraulic press, and pressure transmissibility. |
| **ISAAC NEWTON** (1642–1727)  
Explored various aspects of fluid resistance—inertial, viscous, and wave; discovered jet contraction. |
| **HENRI de PITOT** (1695–1771)  
Constructed double-tube device to indicate water velocity through differential head. |
| **DANIEL BERNOULLI** (1700–1782)  
Experimented and wrote on many phases of fluid motion, coining name "hydromechanics"; devised manometry technique and adapted primitive energy principle to explain velocity-head indication; proposed jet propulsion. |
| **LEONHARD EULER** (1707–1783)  
First explained role of pressure in fluid flow; formulated basic equations of motion and so-called Bernoulli theorem; introduced concept of cavitation and principle of centrifugal machinery. |
| **JEAN le ROND d’ALEMBERT** (1717–1783)  
Originated notion of velocity and acceleration components, differential expression of continuity, and paradox of zero resistance to steady nonuniform motion. |
| **ANTOINE CHEZY** (1718–1798)  
Formulated similarity parameter for predicting flow characteristics of one channel from measurements on another. |
| **GIOVANNI BATTISTA VENTURI** (1746–1822)  
Performed tests on various forms of mouthpieces—in particular, conical contractions and expansions. |
| **LOUIS MARIE HENRI NAVIER** (1785–1836)  
Extended equations of motion to include "molecular" forces. |
| **AUGUSTIN LOUIS de CAUCHY** (1789–1857)  
Contributed to the general field of theoretical hydrodynamics and to the study of wave motion. |
| **GOTTHILF HEINRICH LUDWIG HAGEN** (1797–1884)  
Conducted original studies of resistance in and transition between laminar and turbulent flow. |
| **JEAN LOUIS POISEUILLE** (1799–1869)  
Performed meticulous tests on resistance of flow through capillary tubes. |
| **HENRI PHILIBERT GASPARD DARCY** (1803–1858)  
Performed extensive tests on filtration and pipe resistance; initiated open-channel studies carried out by Bazin. |
| **JULIUS WEISBACH** (1806–1871)  
Incorporated hydraulics in treatise on engineering mechanics, based on original experiments; noteworthy for flow patterns, nondimensional coefficients, weir, and resistance equations. |
| **WILLIAM FROUDE** (1810–1879)  
Developed many towing-tank techniques, in particular the conversion of wave and boundary layer resistance from model to prototype scale. |
| **ROBERT MANNING** (1816–1897)  
Proposed several formulas for open-channel resistance. |
| **GEORGE GABRIEL STOKES** (1819–1903)  
Derived analytically various flow relationships ranging from wave mechanics to viscous resistance—particularly that for the settling of spheres. |
| **ERNST MACH** (1838–1916)  
One of the pioneers in the field of supersonic aerodynamics. |
| **OSBORNE REYNOLDS** (1842–1912)  
Described original experiments in many fields—cavitation, river model similarity, pipe resistance—and devised two parameters for viscous flow; adapted equations of motion of a viscous fluid to mean conditions of turbulent flow. |
| **JOHN WILLIAM STRUTT, LORD RAYLEIGH** (1842–1919)  
Investigated hydrodynamics of bubble collapse, wave motion, jet instability, laminar flow analogies, and dynamic similarity. |
| **VINCENZ STROHAL** (1850–1922)  
Investigated the phenomenon of "singing wires."** |
**TABLE 1.9 (continued)**

<table>
<thead>
<tr>
<th>Name</th>
<th>Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>EDGAR BUCKINGHAM (1867–1940)</td>
<td>Stimulated interest in the United States in the use of dimensional analysis.</td>
</tr>
<tr>
<td>MORITZ WEBER (1871–1951)</td>
<td>Emphasized the use of the principles of similitude in fluid flow studies and formulated a capillarity similarity parameter.</td>
</tr>
<tr>
<td>LUDWIG PRANDTL (1875–1953)</td>
<td>Introduced concept of the boundary layer and is generally considered to be the father of present-day fluid mechanics.</td>
</tr>
<tr>
<td>LEWIS FERRY MOODY (1880–1953)</td>
<td>Provided many innovations in the field of hydraulic machinery. Proposed a method of correlating pipe resistance data which is widely used.</td>
</tr>
<tr>
<td>THEODOR VON KÁRMÁN (1881–1963)</td>
<td>One of the recognized leaders of twentieth century fluid mechanics. Provided major contributions to our understanding of surface resistance, turbulence, and wake phenomena.</td>
</tr>
<tr>
<td>PAUL RICHARD HEINRICH BLASIUS (1883–1970)</td>
<td>One of Prandtl’s students who provided an analytical solution to the boundary layer equations. Also, demonstrated that pipe resistance was related to the Reynolds number.</td>
</tr>
</tbody>
</table>

*Adapted from Ref. 2; used by permission of the Iowa Institute of Hydraulic Research, The University of Iowa.

It is, of course, impossible to summarize the rich history of fluid mechanics in a few paragraphs. Only a brief glimpse is provided, and we hope it will stir your interest. References 2 to 5 are good starting points for further study, and in particular Ref. 2 provides an excellent, broad, easily read history. Try it—you might even enjoy it!

**References**


**Review Problems**

**Note:** Problems designated with (R) are review problems. The phrases within parentheses refer to the main topics to be used in solving the problems. Complete, detailed solutions to these review problems can be found in the supplement titled *Student Solution Manual for Fundamentals of Fluid Mechanics*, by Munson, Young, and Okiishi (John Wiley and Sons, New York, 1997).

1.1R (Dimensions) During a study of a certain flow system the following equation relating the pressures $p_1$ and $p_2$ at two points was developed:

$$p_2 = p_1 + \frac{fE\nu}{D_g}$$

In this equation $\nu$ is a velocity, $E$ the distance between the two points, $D$ a diameter, $g$ the acceleration of gravity, and $f$ a dimensionless coefficient. Is the equation dimensionally consistent?

(ANS: No)
1.2R (Dimensions) If \( V \) is a velocity, \( \ell \) a length, \( W \) a weight, and \( \mu \) a fluid property having dimensions of \( FL^{-2}T \), determine the dimensions of: (a) \( V^2W/\mu \), (b) \( W\mu\ell \), (c) \( V\mu/\ell \), and (d) \( V^2\mu/W \).

(ANS: \( L^{3}T^{-1} \), \( F^{2}L^{-1}T \), \( FL^{-2}L \))

1.3R (Units) Make use of Table 1.4 to express the following quantities in SI units: (a) 465 W, (b) 92.1 J, (c) 536 N/m², (d) 85.9 mm², (e) 386 kg/m².

(ANS: \( 3.43 \times 10^{2} \) ft·lb/s; 67.9 ft·lb; 11.2 lb/ft²; \( 3.03 \times 10^{-6} \) ft³; 2.46 slugs/ft²)

1.4R (Units) A person weighs 165 lb at the earth’s surface. Determine the person’s mass in slugs, kilograms, and pounds mass.

(ANS: 5.12 slugs; 74.8 kg; 165 lb)

1.5R (Specific gravity) Make use of Fig. 1.1 to determine the specific gravity of water at 22 and 89°F. What is the specific volume of water at these two temperatures?

(ANS: 0.998; 0.966; \( 1.002 \times 10^{-3} \) m³/kg)

1.6R (Specific weight) A 1-ft diameter cylindrical tank that is 5 ft long weighs 125 lb and is filled with a liquid having a specific weight of 69.6 lb/ft³. Determine the vertical force required to give the tank an upward acceleration of 9 ft/s².

(ANS: 509 lb up)

1.7R (Ideal gas law) Calculate the density and specific weight of air at a gage pressure of 100 psi and a temperature of 100 °F. Assume standard atmospheric pressure.

(ANS: \( 1.72 \times 10^{-3} \) slugs/ft³; 0.554 lb/ft³)

1.8R (Ideal gas law) A large dirigible having a volume of 90,000 m³ contains helium under standard atmospheric conditions [pressure = 101 kPa (abs) and temperature = 15 °C]. Determine the density and total weight of the helium.

(ANS: 0.169 kg/m³; 1.49 \( \times 10^{3} \) N)

1.9R (Viscosity) A Newtonian fluid having a specific gravity of 0.92 and a kinematic viscosity of \( 4 \times 10^{-4} \) m²/s flows past a fixed surface. The velocity profile near the surface is shown in Fig. P1.9R. Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of \( U \) and \( \delta \), with \( U \) and \( \delta \) expressed in units of meters per second and meters, respectively.

(ANS: 0.578 U/ \( \delta \) m² acting to right on plate)

1.10R (Viscosity) A large movable plate is located between two large fixed plates as shown in Fig. P1.10R. Two Newtonian fluids having the viscosities indicated are contained between the plates. Determine the magnitude and direction of the shearing stresses that act on the fixed walls when the moving plate has a velocity of 4 m/s as shown. Assume that the velocity distribution between the plates is linear.

(ANS: 13.3 N/m² in direction of moving plate)

![Figure P1.10R](image)

1.11R (Viscosity) Determine the torque required to rotate a 50-mm-diameter vertical cylinder at a constant angular velocity of 30 rad/s inside a fixed outer cylinder that has a diameter of 50.2 mm. The gap between the cylinders is filled with SAE 10 oil at 20 °C. The length of the inner cylinder is 200 mm. Neglect bottom effects and assume the velocity distribution in the gap is linear. If the temperature of the oil increases to 80 °C, what will be the percentage change in the torque?

(ANS: \( 0.589 \) N m; 92.0 percent)

1.12R (Bulk modulus) Estimate the increase in pressure (in psi) required to decrease a unit volume of mercury by 0.1%.

(ANS: \( 4.14 \times 10^{3} \) psi)

1.13R (Bulk modulus) What is the isothermal bulk modulus of nitrogen at a temperature of 90 °F and an absolute pressure of 5600 lb/ft²?

(ANS: 5600 lb/ft²)

1.14R (Speed of sound) Compare the speed of sound in mercury and oxygen at 20 °C.

(ANS: \( c_{Hg}/c_{O2} = 4.45 \))

1.15R (Vapor pressure) At a certain altitude it was found that water boils at 90 °C. What is the atmospheric pressure at this altitude?

(ANS: 70.1 kPa (abs))
Problems

Note: Unless specific values of required fluid properties are given in the statement of the problem, use the values found in the tables on the inside of the front cover. Problems designated with an (*) are intended to be solved with the aid of a programmable calculator or a computer. Problems designated with a (†) are "open-ended" problems and require critical thinking in that to work them one must make various assumptions and provide the necessary data. There is not a unique answer to these problems.

1.1 Determine the dimensions, in both the FLT system and the MLT system, for (a) the product of mass times velocity, (b) the product of force times volume, and (c) kinetic energy divided by area.

1.2 Verify the dimensions, in both the FLT and MLT systems, of the following quantities which appear in Table 1.1: (a) angular velocity, (b) energy, (c) moment of inertia (area), (d) power, and (e) pressure.

1.3 Verify the dimensions, in both the FLT system and the MLT system, of the following quantities which appear in Table 1.1: (a) acceleration, (b) stress, (c) moment of a force, (d) volume, and (e) work.

1.4 If \( P \) is a force and \( x \) a length, what are the dimensions (in the FLT system) of (a) \( dP/\,dx \), (b) \( d^2P/\,dx^2 \), and (c) \( f \,dP/\,dx \)?

1.5 If \( u \) is a velocity, \( x \) a length, and \( t \) a time, what are the dimensions (in the MLT system) of (a) \( \partial u/\,\partial t \), (b) \( \partial^2 u/\,\partial x\,\partial t \), and (c) \( \int (\partial u/\,\partial t) \,\,dx \)?

1.6 If \( V \) is a velocity, \( \ell \) a length, and \( \nu \) a fluid property having dimensions of \( L^2T^{-1} \), which of the following combinations are dimensionless: (a) \( V\ell^2/\nu \), (b) \( V\ell/\nu \), (c) \( V^2\nu \), (d) \( V/\ell\nu \)?

1.7 Dimensionless combinations of quantities (commonly called dimensionless parameters) play an important role in fluid mechanics. Make up five possible dimensionless parameters by using combinations of some of the quantities listed in Table 1.1.

1.8 The force, \( P \), that is exerted on a spherical particle moving slowly through a liquid is given by the equation

\[ P = 3\pi \mu DV \]

where \( \mu \) is a fluid property (viscosity) having dimensions of \( FL^{-2}T \), \( D \) is the particle diameter, and \( V \) is the particle velocity. What are the dimensions of the constant, \( 3\pi \)? Would you classify this equation as a general homogeneous equation?

1.9 According to information found in an old hydraulics book, the energy loss per unit weight of fluid flowing through a nozzle connected to a hose can be estimated by the formula

\[ h = (0.04 \text{ to } 0.09) (D/d)^{4/3} V^2/2g \]

where \( h \) is the energy loss per unit weight, \( D \) the hose diameter, \( d \) the nozzle tip diameter, \( V \) the fluid velocity in the hose, and \( g \) the acceleration of gravity. Do you think this equation is valid in any system of units? Explain.

1.10 The pressure difference, \( \Delta p \), across a partial blockage in an artery (called a stenosis) is approximated by the equation

\[ \Delta p = \frac{\mu V}{D} + K_u \left( \frac{A_0}{A_1} - 1 \right)^2 \rho V^2 \]

where \( V \) is the blood velocity, \( \mu \) the blood viscosity \((FL^{-2}T)\), \( \rho \) the blood density \((ML^{-3})\), \( D \) the artery diameter, \( A_0 \) the area of the unobstructed artery, and \( A_1 \) the area of the stenosis. Determine the dimensions of the constants \( K_u \) and \( K_u \). Would this equation be valid in any system of units?

1.11 Assume that the speed of sound, \( c \), in a fluid depends on an elastic modulus, \( E \), with dimensions \( FL^{-2} \), and the fluid density, \( \rho \), in the form \( c = (E \rho /\rho)^{1/2} \). If this is to be a dimensionally homogeneous equation, what are the values for \( a \) and \( b \)? Is your result consistent with the standard formula for the speed of sound? (See Eq. 1.19.)

1.12 A formula to estimate the volume rate of flow, \( Q \), flowing over a dam of length, \( B \), is given by the equation

\[ Q = 3.09BH^{1/2} \]

where \( H \) is the depth of the water above the top of the dam (called the head). This formula gives \( Q \) in \( ft^3/s \) when \( B \) and \( H \) are in feet. Is the constant 3.09 dimensionless? Would this equation be valid if units other than feet and seconds were used?

† 1.13 Cite an example of a restricted homogeneous equation contained in a technical article found in an engineering journal in your field of interest. Define all terms in the equation, explain why it is a restricted equation, and provide a complete journal citation (title, date, etc.).

1.14 Make use of Table 1.3 to express the following quantities in SI units: (a) 10.2 in./min., (b) 4.81 slugs, (c) 3.02 lb, (d) 73.1 ft/s, (e) 0.0234 lb-ft/s.

1.15 Make use of Table 1.4 to express the following quantities in BQ units: (a) 14.2 km, (b) 8.14 N/m³, (c) 1.61 kg/m³, (d) 0.0320 N·m/s, (e) 5.67 mm/hr.

1.16 Make use of Appendix A to express the following quantities in SI units: (a) 160 acre, (b) 742 Bu, (c) 240 miles, (d) 79.1 hp, (e) 60.3 °F.

1.17 Verify the conversion relationships for (a) acre, (b) bar, and (c) U.S. liquid gallon found in Appendix A.

1.18 For Table 1.3 verify the conversion relationships for: (a) area, (b) density, (c) velocity, and (d) specific weight. Use the basic conversion relationships: 1 ft = 0.3048 m; 1 lb = 4.4482 N; and 1slug = 14.594 kg.

1.19 For Table 1.4 verify the conversion relationships for: (a) acceleration, (b) density, (c) pressure, and (d) volume flowrate. Use the basic conversion relationships: 1 m = 3.2808 ft; 1 N = 0.22481 lb; and 1 kg = 0.068521 slug.

1.20 Water flows from a large drainage pipe at a rate of 1500 gal/min. What is this volume rate of flow in m³/s and in liters/min?

1.21 A tank of oil has a mass of 30 slugs. (a) Determine its weight in pounds and in newtons at the earth's surface. (b) What would be its mass (in slugs) and its weight (in pounds) if located