CEM Part II: Chapter 1
Water Wave Mechanics

- Regular Waves
    - Wave Theories
    - Useful Relationships

- Irregular Waves
    - Spectral Analysis

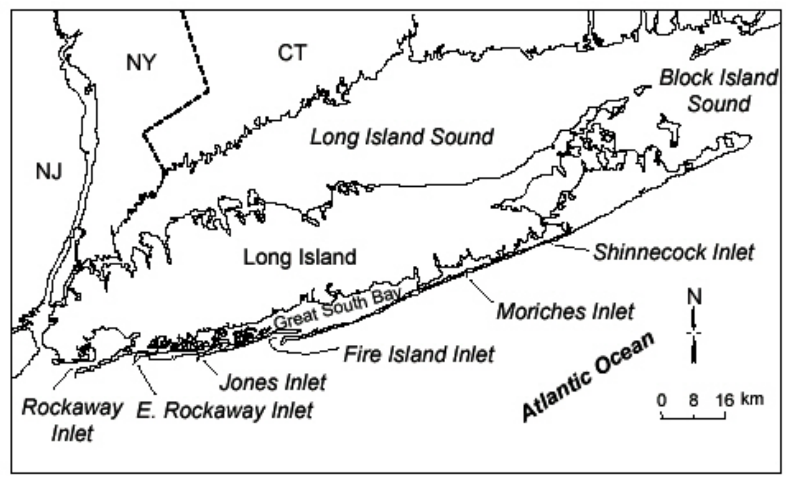


Fig. 1. Location map for the south shore of Long Island, New York

II-1-1. Introduction
Surface Gravity Waves *on the ocean with periods of 3 to 25 sec* (internal waves, tides, and edge waves)
a.Disturbing force = wind   Restoring force = gravity

The Regular Waves section *of this chapter begins with the simplest mathematical representation* assuming ocean waves are two-dimensional (2-D), small in amplitude, sinusoidal, and progressively definable by their wave height and period in a given water depth.

These theories become nonlinear and allow formulation of waves that are not of purely sinusoidal in shape; for example, waves having the flatter troughs and peaked crests typically seen in shallow coastal waters when waves are relatively high.

The Irregular Waves *section of this chapter is devoted to an alternative description of ocean waves.* Statistical methods for describing the natural time-dependent three-dimensional characteristics of real wave systems are presented.

One approach is to transform the sea surface using Fourier theory into summation of simple sine waves and then to define a wave's characteristics in terms of its spectrum. This allows treatment of the variability of waves with respect to period and direction of travel.

The second approach is to describe a wave record at a point as a sequence of individual waves with different heights and periods and then to consider the variability of the wave field in terms of the probability of individual waves.

General Notes:

The major generating force for waves is the wind acting on the air-sea interface.

A significant amount of wave energy is dissipated in the nearshore region and on beaches. **Wave energy forms beaches; sorts bottom sediments on the shore face; transports bottom materials onshore, offshore, and alongshore; and exerts forces upon coastal structures.**

The Regular Waves section of this chapter provides only an introduction to wave mechanics, and it focuses on simple water wave theories for coastal engineers.

Methods are discussed for estimating wave surface profiles, water particle motion, wave energy, and wave transformations due to interaction with the bottom and with structures.

General Notes:

•The simplest wave theory is the *first-order, small-amplitude, or* **Airy wave theory** *which will hereafter* be called **linear theory**.

•*Many engineering problems can be handled with ease and reasonable accuracy by* this theory.

•*When waves become large or travel toward shore into shallow water,* **higher-order wave theories** *are* often required to describe wave phenomena. These theories represent **nonlinear waves**.

Wave Theories

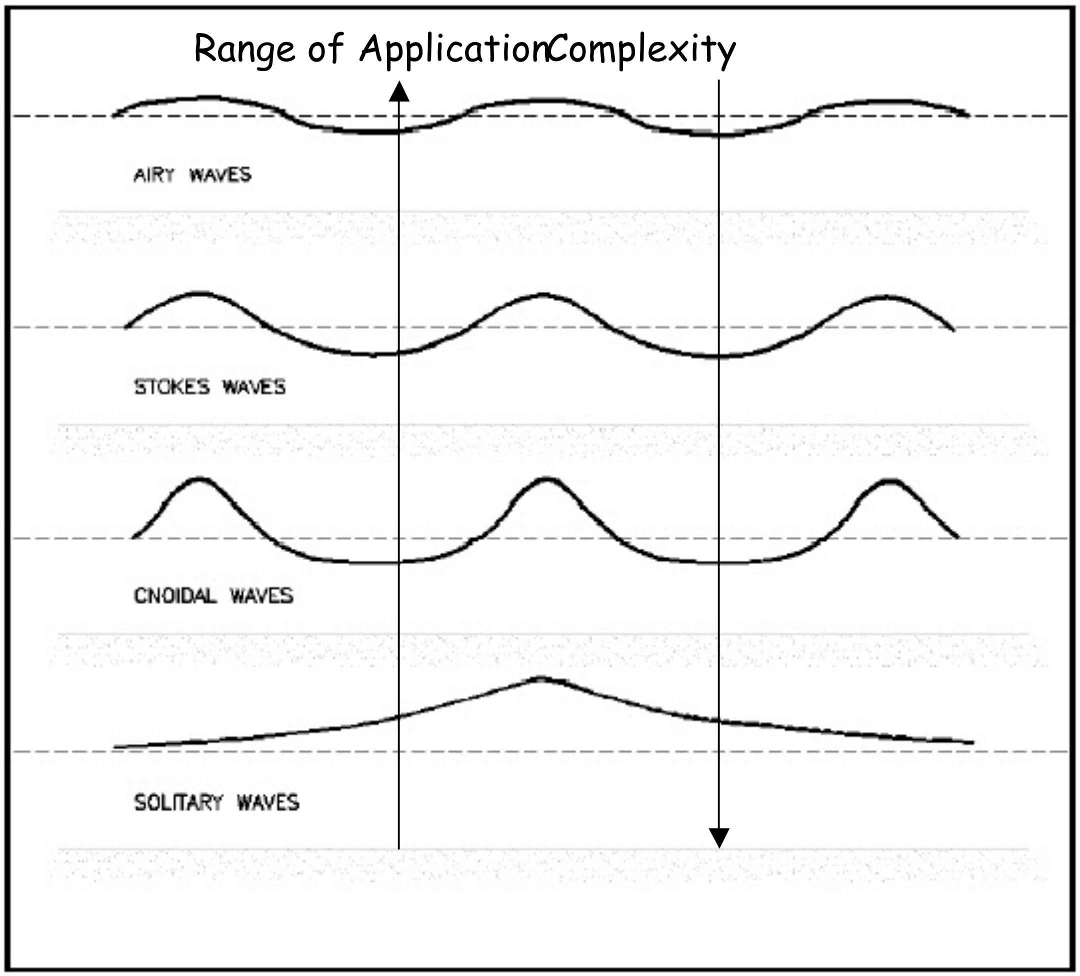
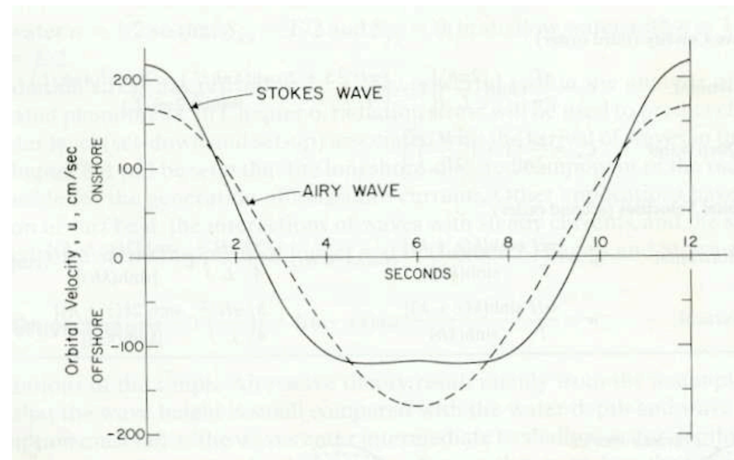
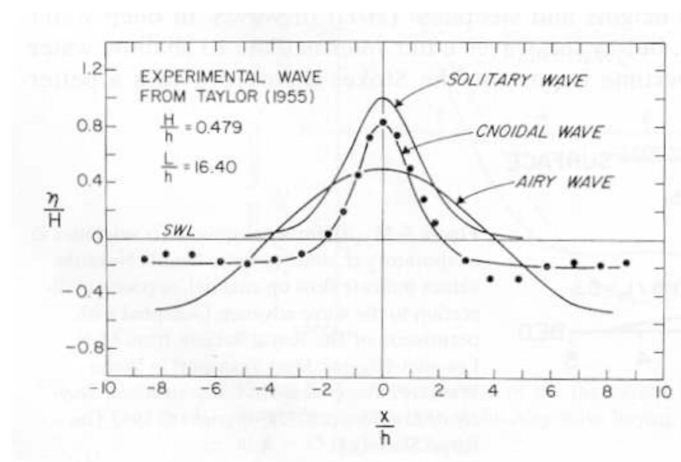


Figure II-1-10. Wave profile shape of different progressive gravity waves





Any basic physical description of a water wave involves both its **surface form** and the **water motion** beneath the surface. A wave that can be described in simple mathematical terms is called a *simple wave.*

Waves comprised of several components and difficult to describe in form or motion are termed **wave trains or complex waves**.

Sinusoidal or monochromatic waves are examples of simple waves, since their surface profile can be described by a single sine or cosine function.

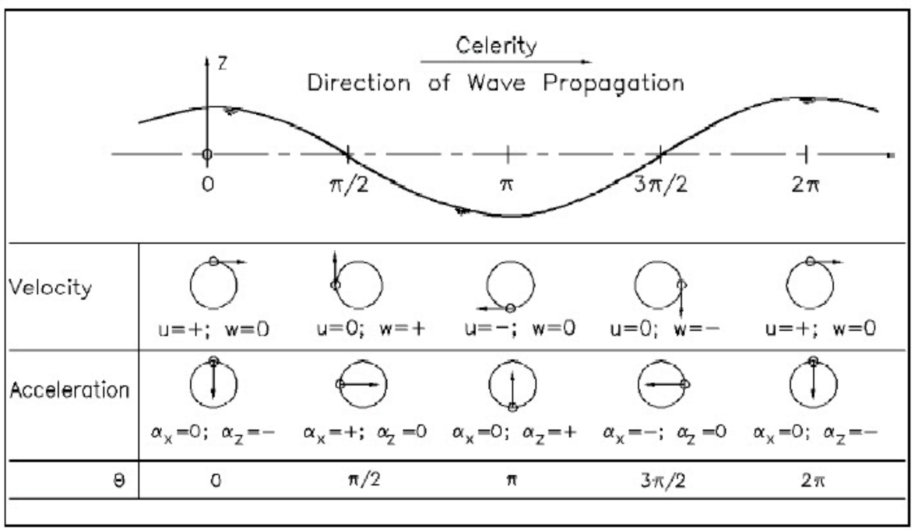
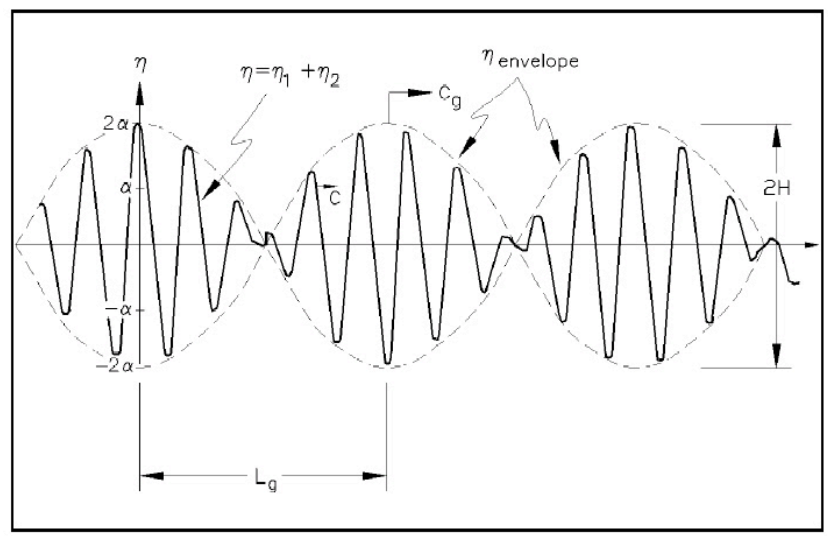


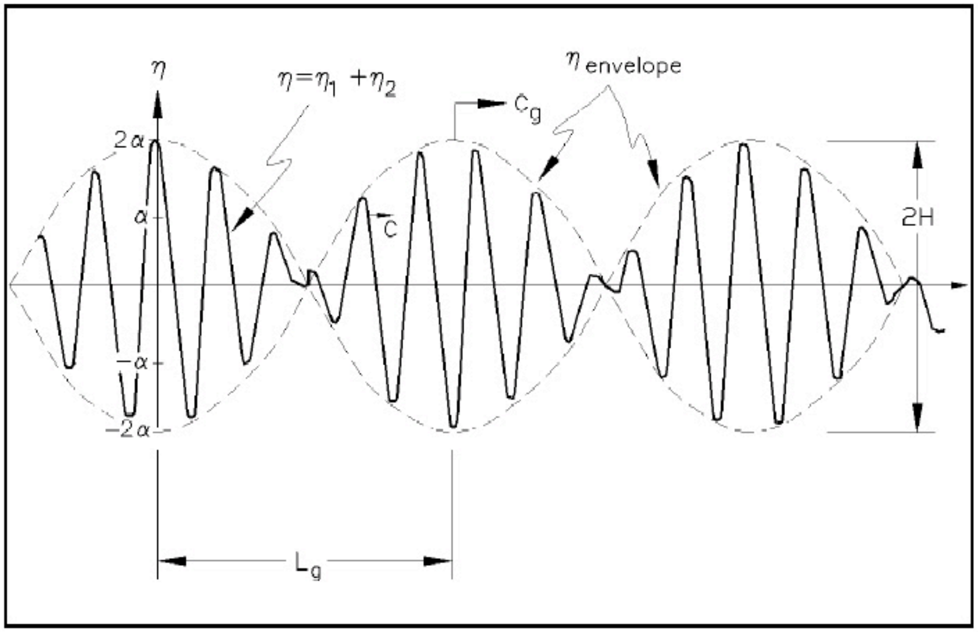
Figure II-1-2. Local fluid velocities and accelerations



A wave is *periodic if its motion and surface* profile recur in equal intervals of time termed the *wave period.*

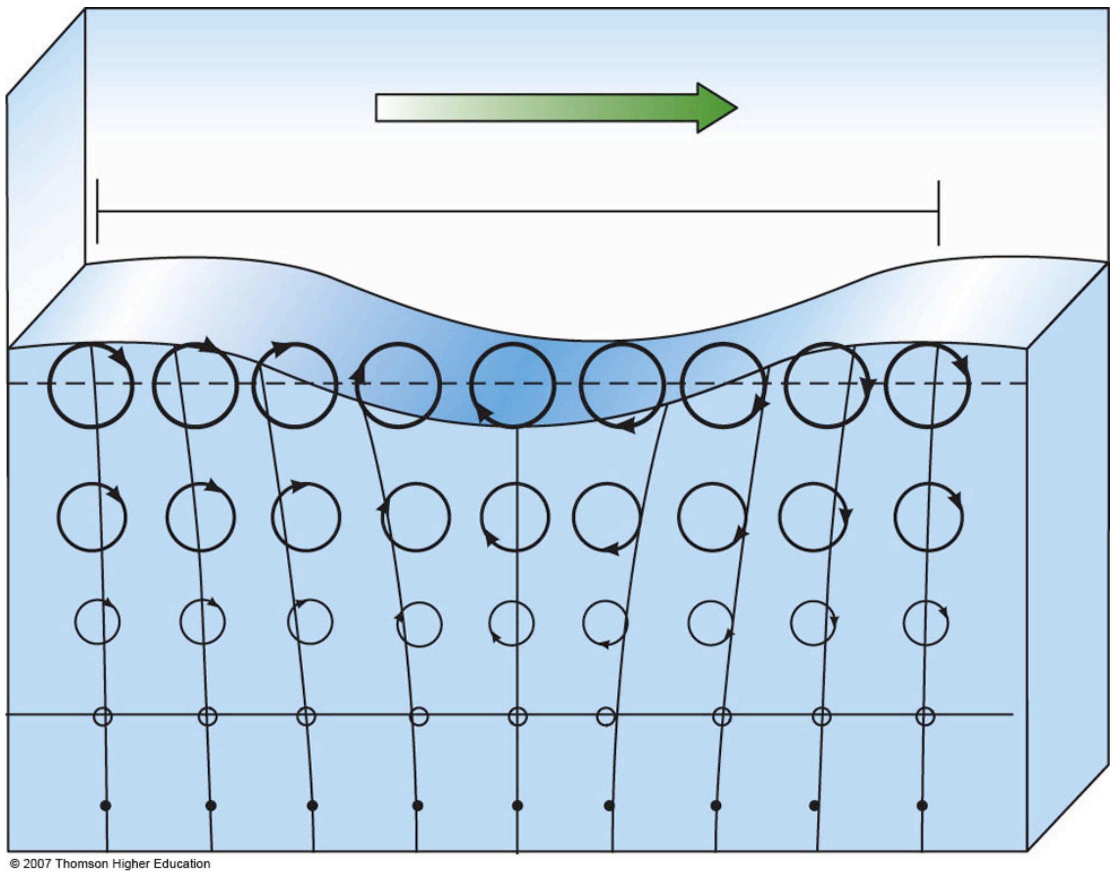
A wave form that moves horizontally relative to a fixed point is called a **progressive wave** *and the direction in which it moves is termed the direction of* **wave propagation**.

A progressive wave is called **wave of permanent form** *if it propagates without* experiencing any change in shape.



Water waves are considered *oscillatory or nearly oscillatory if the motion described by the water* particles is circular orbits that are closed or nearly closed for each wave period.

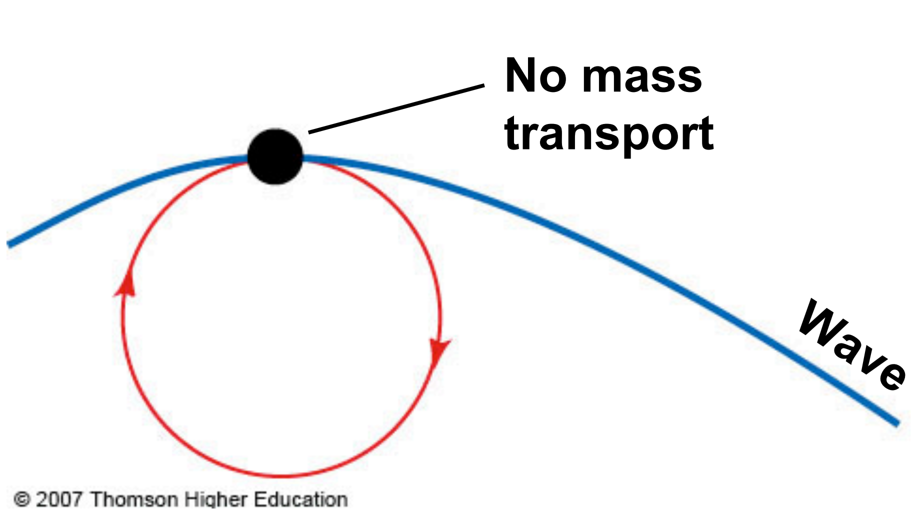
The linear theory represents pure oscillatory waves.



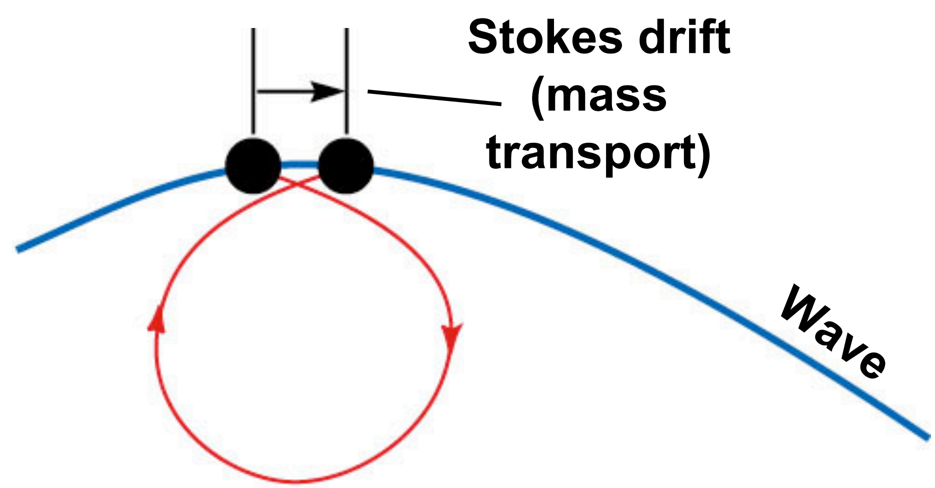
Waves defined by finite-amplitude wave theories are not pure oscillatory waves but still periodic since the fluid is moved in the direction of wave advance by each successive wave.

This motion is termed *mass transport of the waves. When water particles advance with the wave and do not return to their* original position, the wave is called a *wave of translation.*

A solitary wave is an example of a wave of translation.



Closed orbit after one period

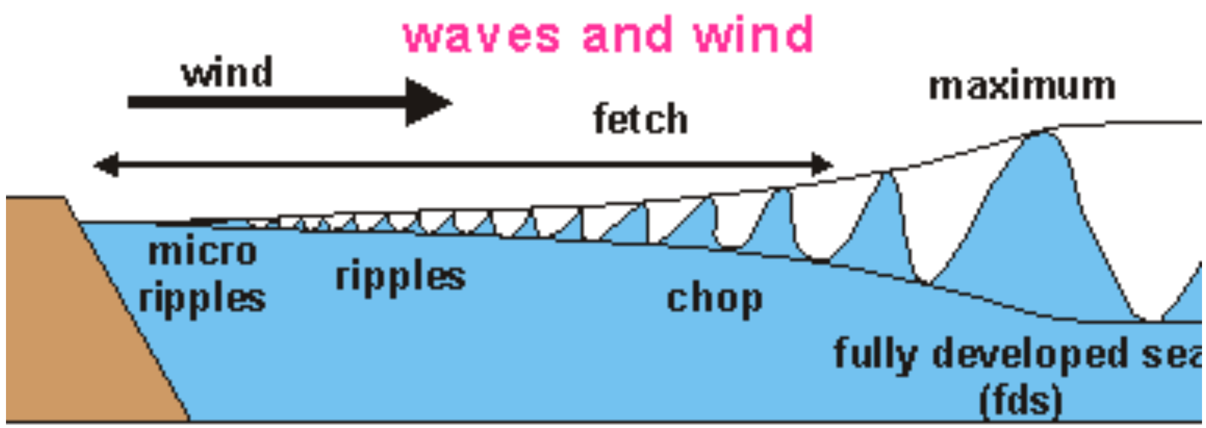


Open orbit after one period

Seas and Swells.

Seas refer to short-period waves still being created by winds.

Swells refer to waves that have moved out of the generating area. In general, swells are more regular waves with well-defined long crests and relatively long periods.



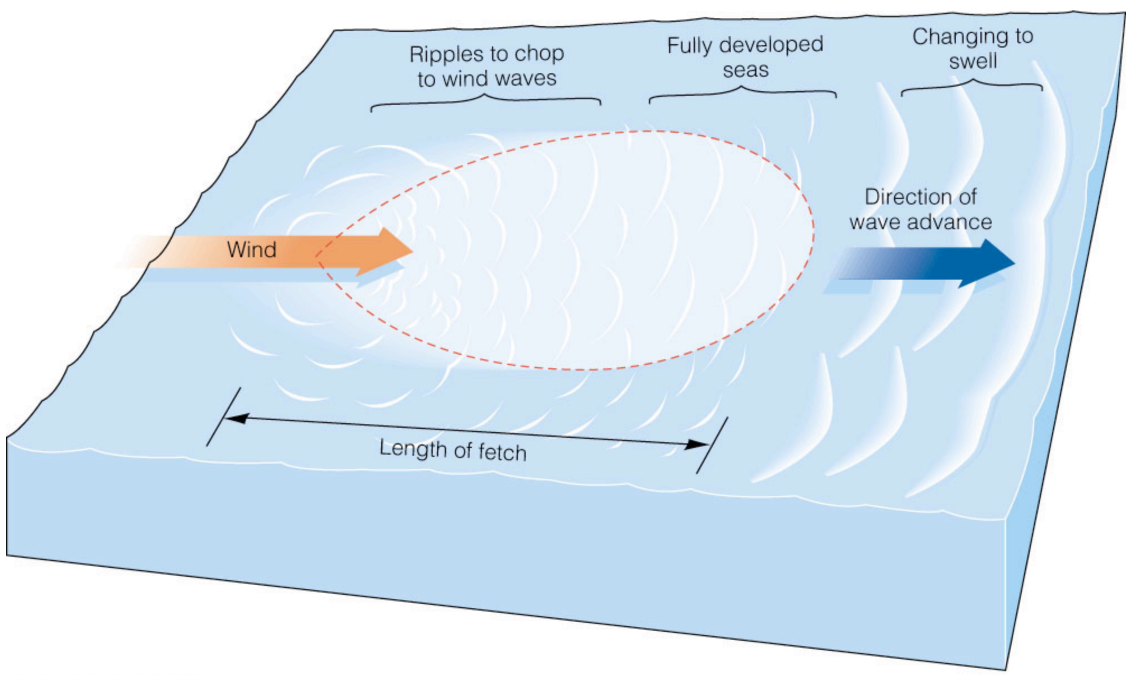
As waves develop, they offer more surface area for the wind to press against (wind stress). Depending on both fetch and time, the size of the waves increases quadratically to a maximum. The energy imparted to the sea increases with the fourth power of the wind speed! As waves develop, they become more rounded and longer and they travel faster. Their maximum size is reached when they travel almost as fast as the wind. A 60 knot storm lasting for 10 hours makes 15m high waves in open water.

The point when waves stop growing is termed a *fully developed sea condition.*

Wind energy imparted to water is dissipated by wave breaking

Seas are short-crested and irregular and their periods are within the 3- to 25-sec range. Seas usually have shorter periods and lengths, and their surface appears much more disturbed than for swells.

Waves assume a more orderly state with the appearance of definite crests and troughs when they are no longer under the influence of winds (swell).



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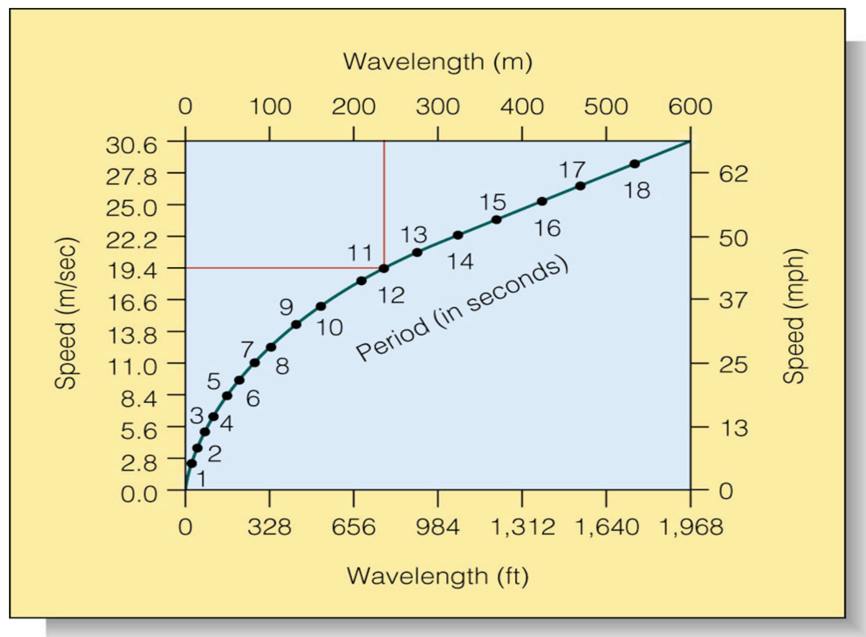
Sea and Swells

Longer period waves move faster and reach distant sites first. Shorter period components may reach the site several days later.

In the wave generation area, energy is transferred from shorter period waves to the longer waves.

Short-period components lose their energy more readily than long-period components.

As a consequence of these processes, the periods of swell waves tend to be somewhat longer than seas. Swells typically have periods greater than 10 sec.



A progressive wave may be represented by the variables *x (spatial) and t (temporal) or by their* combination (phase) $\theta = kx - \omega t$

Often characterized by the wave height *H wavelength L and water depth d.*

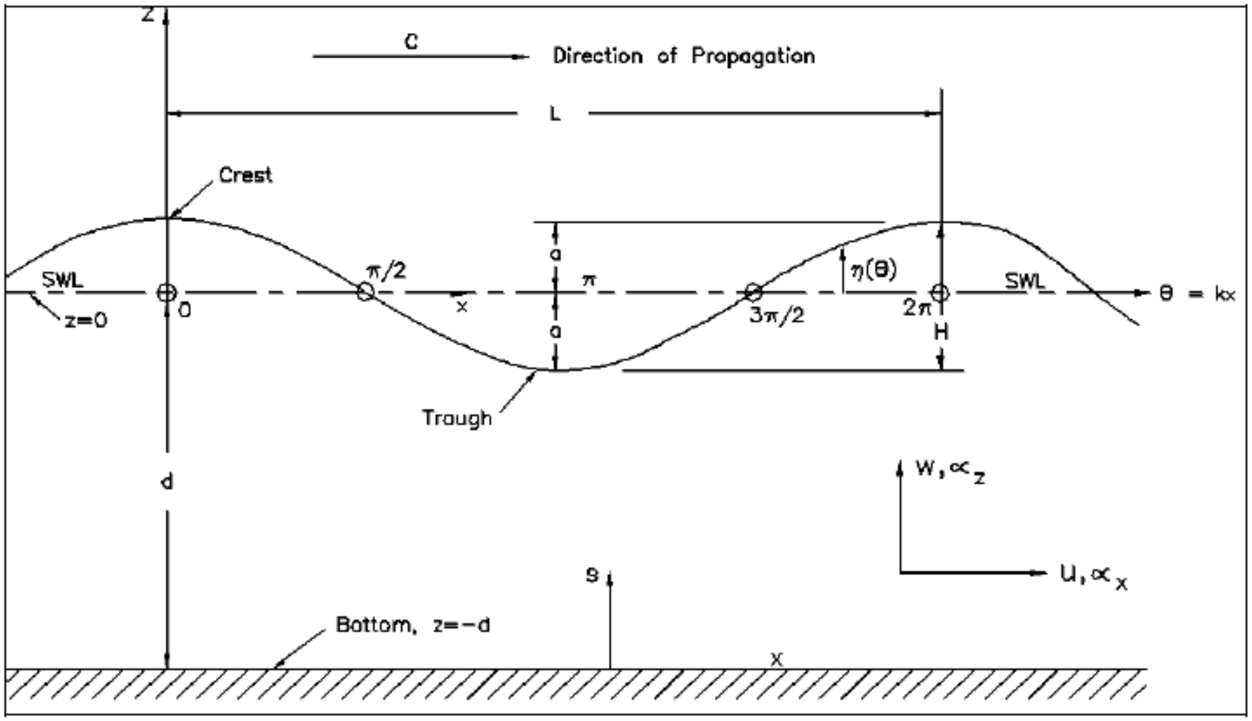


Figure II-1-1. Definition of terms - elementary, sinusoidal, progressive wave

$\omega = 2\pi/T$:radian frequency

$k = 2\pi/L$:wave number

C = L/T, ω/k :celerity

H/L = wave steepness

d/L = relative depth

H/d = relative wave ht

Small-amplitude or Linear-Airy Wave Theory

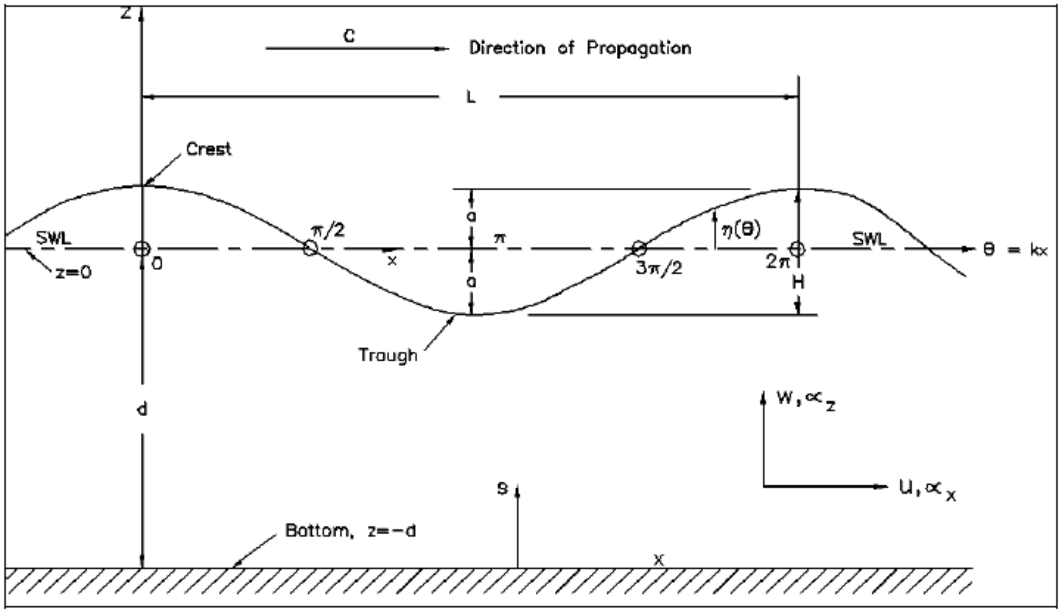


Figure II-1-1. Definition of terms - elementary, sinusoidal, progressive wave

•The fluid is homogeneous and incompressible; density is a constant.

•Surface tension can be neglected.

•Coriolis effect can be neglected.

•Pres. at the free surface is uniform, constant.

•The fluid is ideal or inviscid (lacks viscosity).

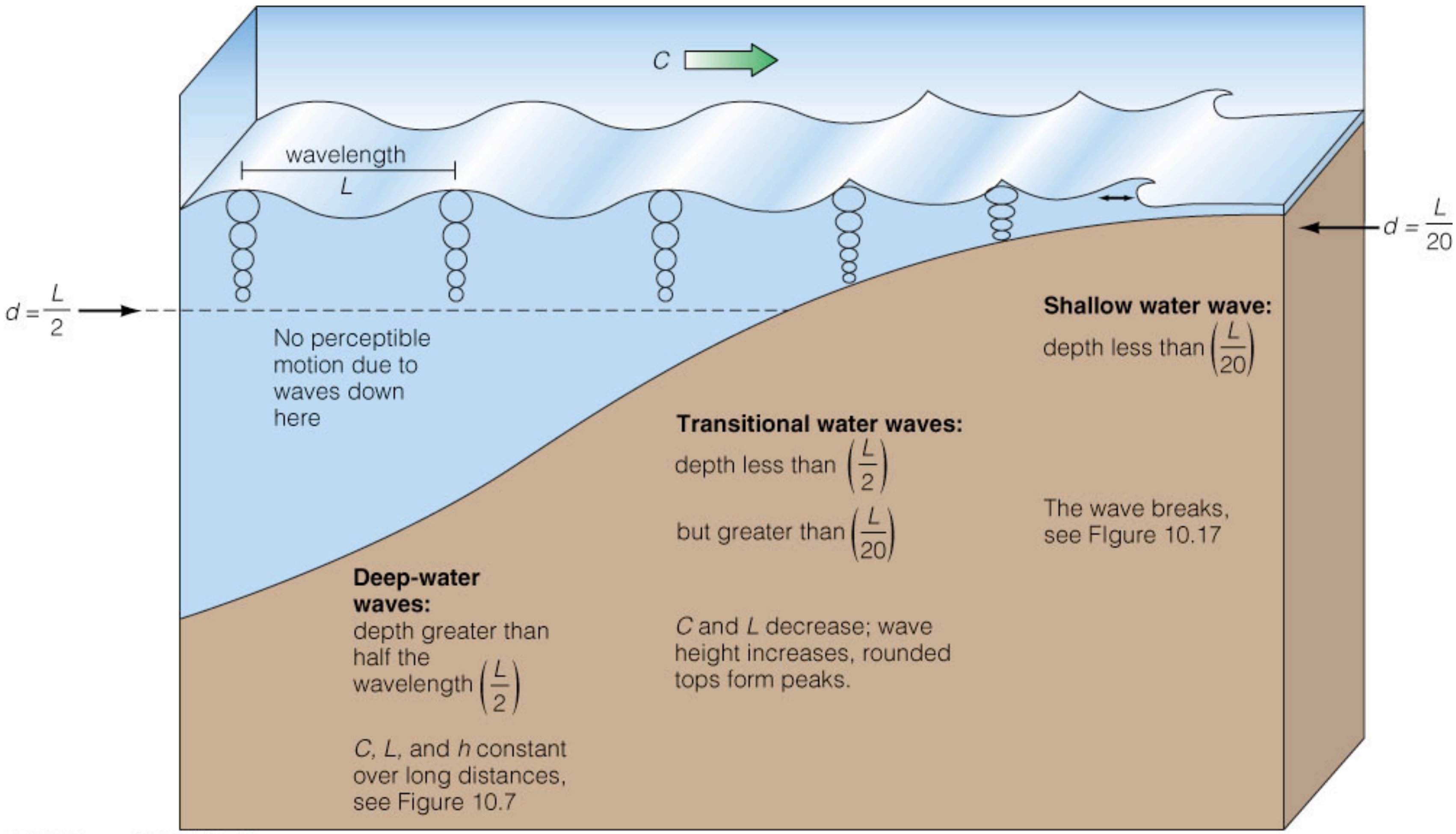
•The particular wave being considered does not interact with any other water motions. The flow is irrotational so that water particles do not rotate (only normal forces are important and shearing forces are negligible).

•The bed is a horizontal, fixed, impermeable boundary, which implies that the vertical velocity at the bed is zero.

•The wave amplitude is small and the waveform is invariant in time and space. ! Waves are plane or long-crested (two-dimensional).

Table II-1-1
Classification of Water Waves

Classification	d/L	kd	tanh (kd)
Deep water	1/2 to ∞	π to ∞	≈1
Transitional	1/20 to 1/2	π/10 to π	tanh (kd)
Shallow water	0 to 1/20	0 to π/10	≈ kd



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$$C = \frac{L}{T}$$

Wave speed or celerity

$$C = \frac{gT}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$$

Dispersion Relationship: relates wave celerity to period, wavelength, and water depth

$$C_0 = \frac{gT}{2\pi}$$

Deep water wave speed is a function of period only

$$C_0 = \frac{gT}{2\pi} = \frac{9.8}{2\pi}T = 1.56T \ m/s$$

$$C = \sqrt{gd}$$

Shallow water wave speed is constrained by water depth

$$L_0 = \frac{gT^2}{2\pi}$$

In deep water wavelength is a function of period. In shallow water it is a function of period and water depth, and must be solved iteratively.

$$L_0 = \frac{gT^2}{2\pi} = \frac{9.8}{2\pi}T^2 = 1.56T^2 \ m$$

Simple Wave Equation

$$\eta = a \cos(kx - \omega t) = \frac{H}{2} \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) = a \cos \theta$$

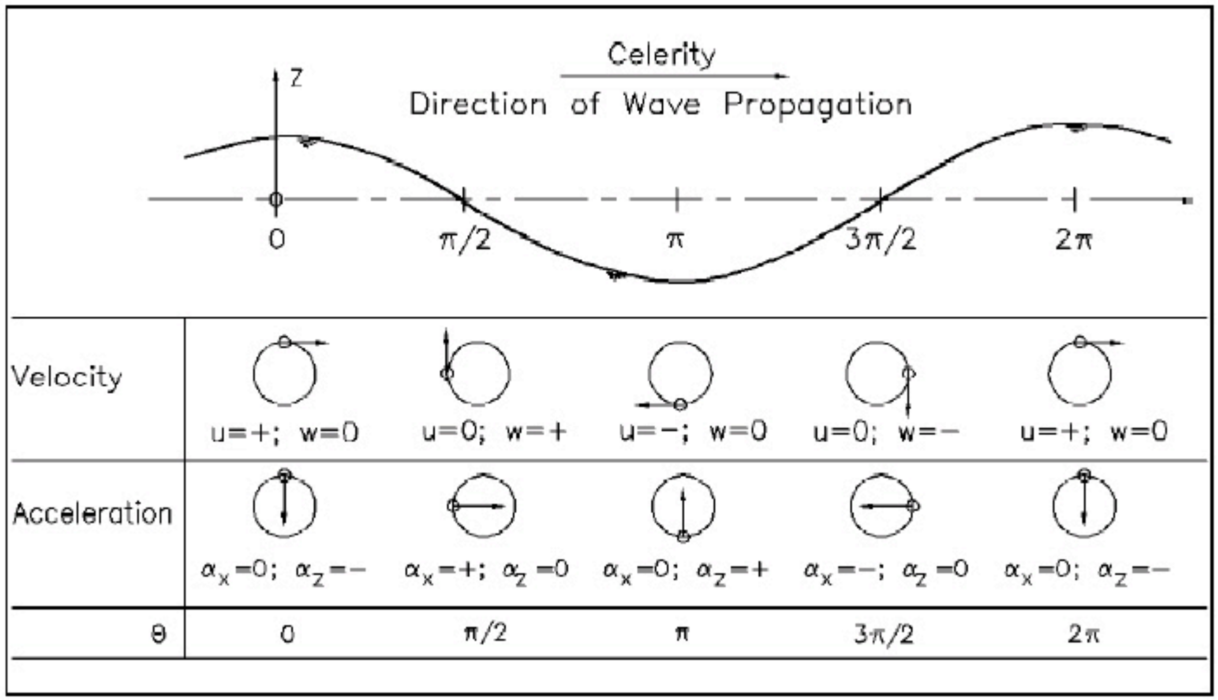


Figure II-1-2. Local fluid velocities and accelerations

$$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$$

Horizontal velocity

$$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$$

Vertical velocity

Water Particle Motion

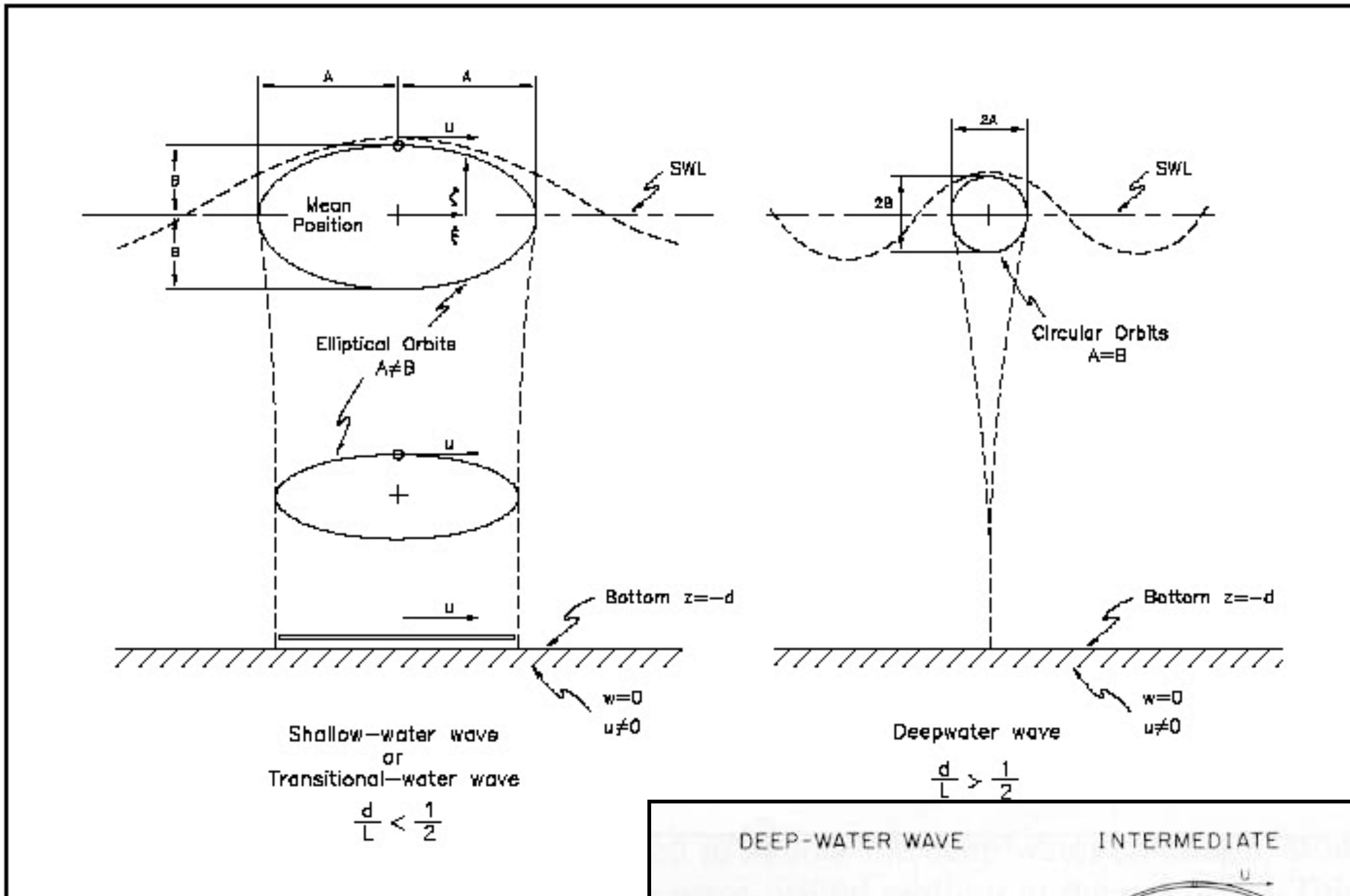
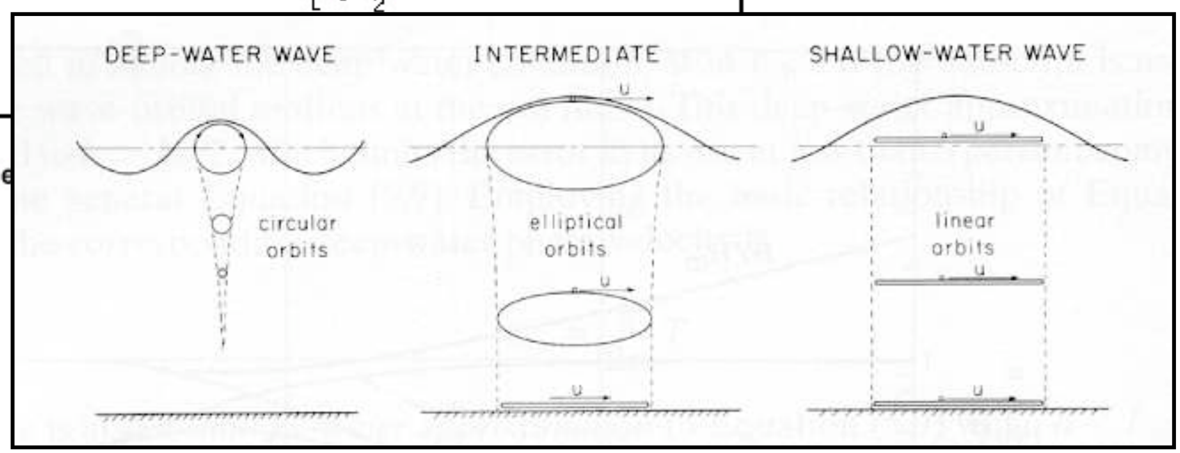


Figure II-1-4. Water particle displacements from me



Wave Propagation
"The superposition of multiple wave trains"

Group Velocity

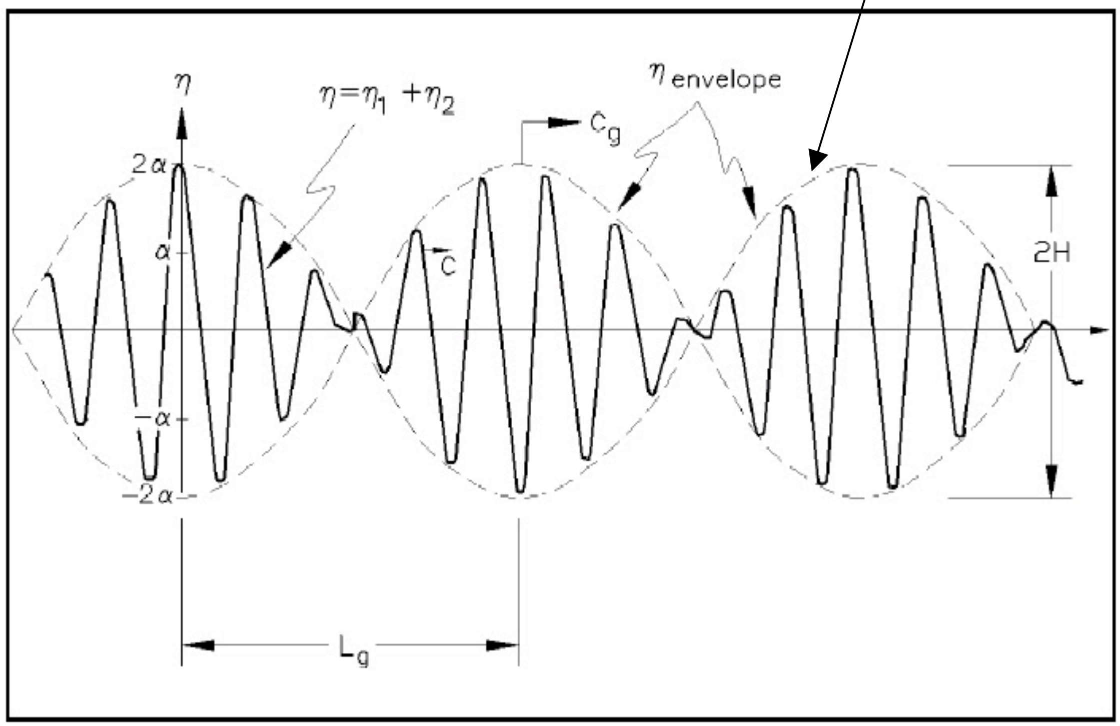
$$C_g = \frac{1}{2}\frac{L}{T}\left[1 + \frac{\frac{4\pi d}{L}}{\sinh\left(\frac{4\pi d}{L}\right)}\right] = nC$$

$$n = \frac{1}{2}\left[1 + \frac{\frac{4\pi d}{L}}{\sinh\left(\frac{4\pi d}{L}\right)}\right]$$

$$C_{g_0} = \frac{1}{2}\frac{L_0}{T} = \frac{1}{2}C_0 \text{ (deep water)}$$

$$C_{g_s} = \frac{L}{T} = C = \sqrt{gd} \quad \text{(shallow water)}$$

Beat Wave



Note: wave energy propagates at the speed of the group which is slower than the speed of the individual wave form in deep water. Once the waves break the beat wave is released and travels as a shallow water wave whose speed is constrained by the water depth.

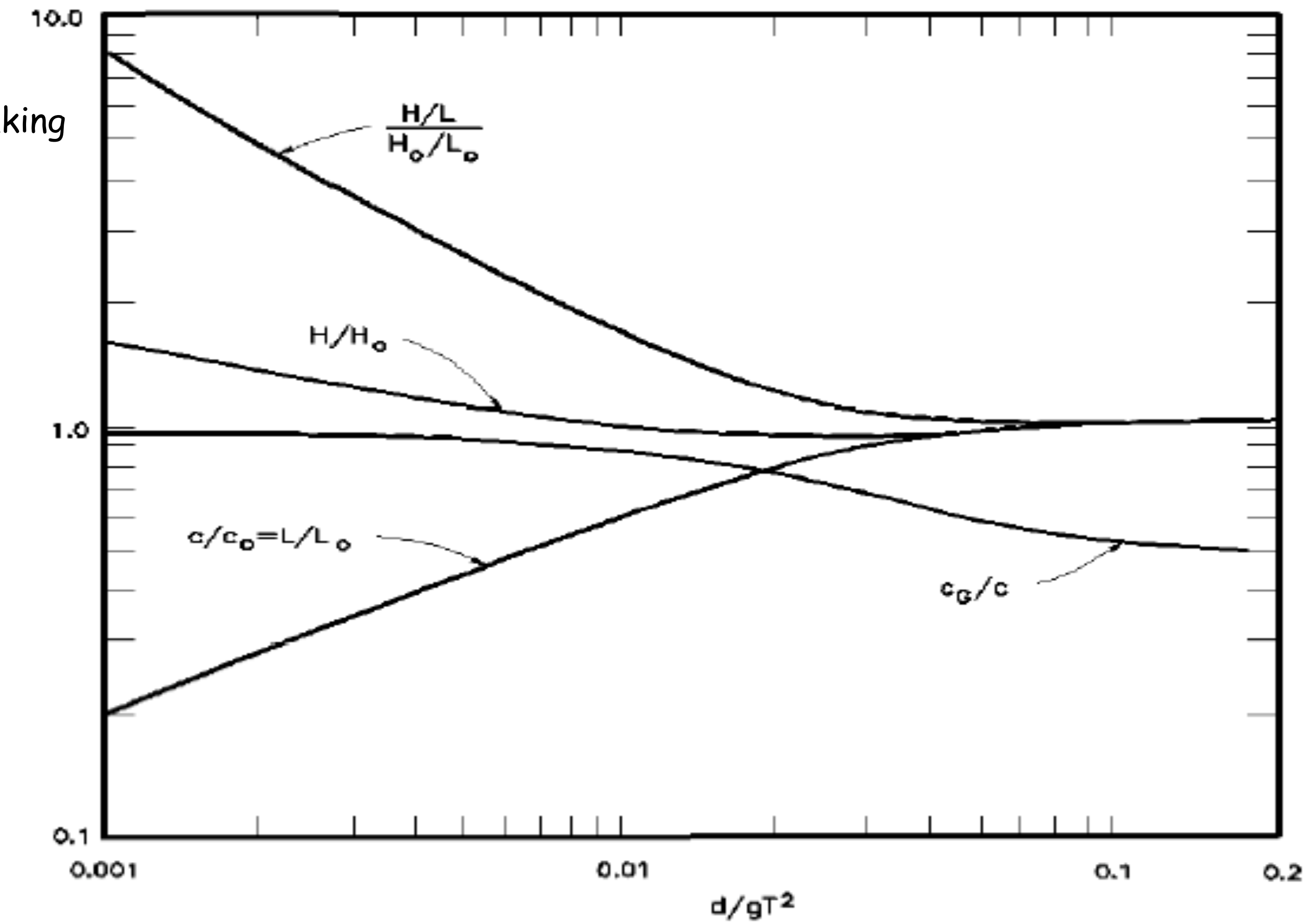
Wave Shoaling: transformation of the wave form due to interaction with bathymetry (intermediate – shallow water)

H: increases up to breaking

T: remains constant

L: decreases

C: decreases



Wave Energy and Power

$$E = E_k + E_p = \frac{\rho g H^2 L}{16} + \frac{\rho g H^2 L}{16} = \frac{\rho g H^2 L}{8}$$

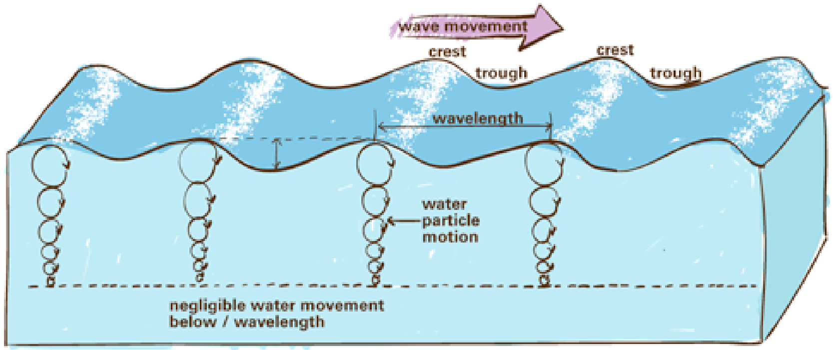
total wave energy in one wavelength per unit crest width

$$\bar{E} = \frac{E}{L} = \frac{\rho g H^2}{8}$$

total average wave energy per unit surface area, *energy density*

$$\bar{P} = \bar{E}nC = \bar{E}C_g$$

Wave power: propagation of energy through the water column



Relative Depth	Shallow Water $\frac{d}{L} < \frac{1}{25}$	Transitional Water $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	Deep Water $\frac{d}{L} < \frac{1}{2}$
1. Wave profile	Same As >	$\eta = \frac{H}{2}\cos\left[\frac{2\pi x}{L} - \frac{2\pi t}{T}\right] = \frac{H}{2}\cos\theta$	< Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right]C$	$C_g = \frac{1}{2}C = \frac{gT}{4\pi}$
5. Water particle velocity (a) Horizontal (b) Vertical	$u = \frac{H}{2}\sqrt{\frac{g}{d}}\cos\theta$ $w = \frac{H\pi}{T}\left(1+\frac{z}{d}\right)\sin\theta$	$u = \frac{H}{2}\frac{gT}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$ $w = \frac{H}{2}\frac{gT}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$	$u = \frac{\pi H}{T}e^{\left(\frac{2\pi z}{L}\right)}\cos\theta$ $w = \frac{\pi H}{T}e^{\left(\frac{2\pi z}{L}\right)}\sin\theta$
6. Water particle accelerations (a) Horizontal (b) Vertical	$a_x = \frac{H\pi}{T}\sqrt{\frac{g}{d}}\sin\theta$ $a_z = -2H\left(\frac{\pi}{T}\right)^2\left(1+\frac{z}{d}\right)\cos\theta$	$a_x = \frac{g\pi H}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$ $a_z = -\frac{g\pi H}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$	$a_x = 2H\left(\frac{\pi}{T}\right)^2 e^{\left(\frac{2\pi z}{L}\right)}\sin\theta$ $a_z = -2H\left(\frac{\pi}{T}\right)^2 e^{\left(\frac{2\pi z}{L}\right)}\cos\theta$
7. Water particle displacements (a) Horizontal (b) Vertical	$\xi = -\frac{HT}{4\pi}\sqrt{\frac{g}{d}}\sin\theta$ $\zeta = \frac{H}{2}\left(1+\frac{z}{d}\right)\cos\theta$	$\xi = -\frac{H}{2}\frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\sin\theta$ $\zeta = \frac{H}{2}\frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\cos\theta$	$\xi = -\frac{H}{2}e^{\left(\frac{2\pi z}{L}\right)}\sin\theta$ $\zeta = \frac{H}{2}e^{\left(\frac{2\pi z}{L}\right)}\cos\theta$

Break Point: point at which wave form becomes unstable and breaks (water particles at the crest travel much faster/farther than water particles in the trough, **depth or steepness induced**)

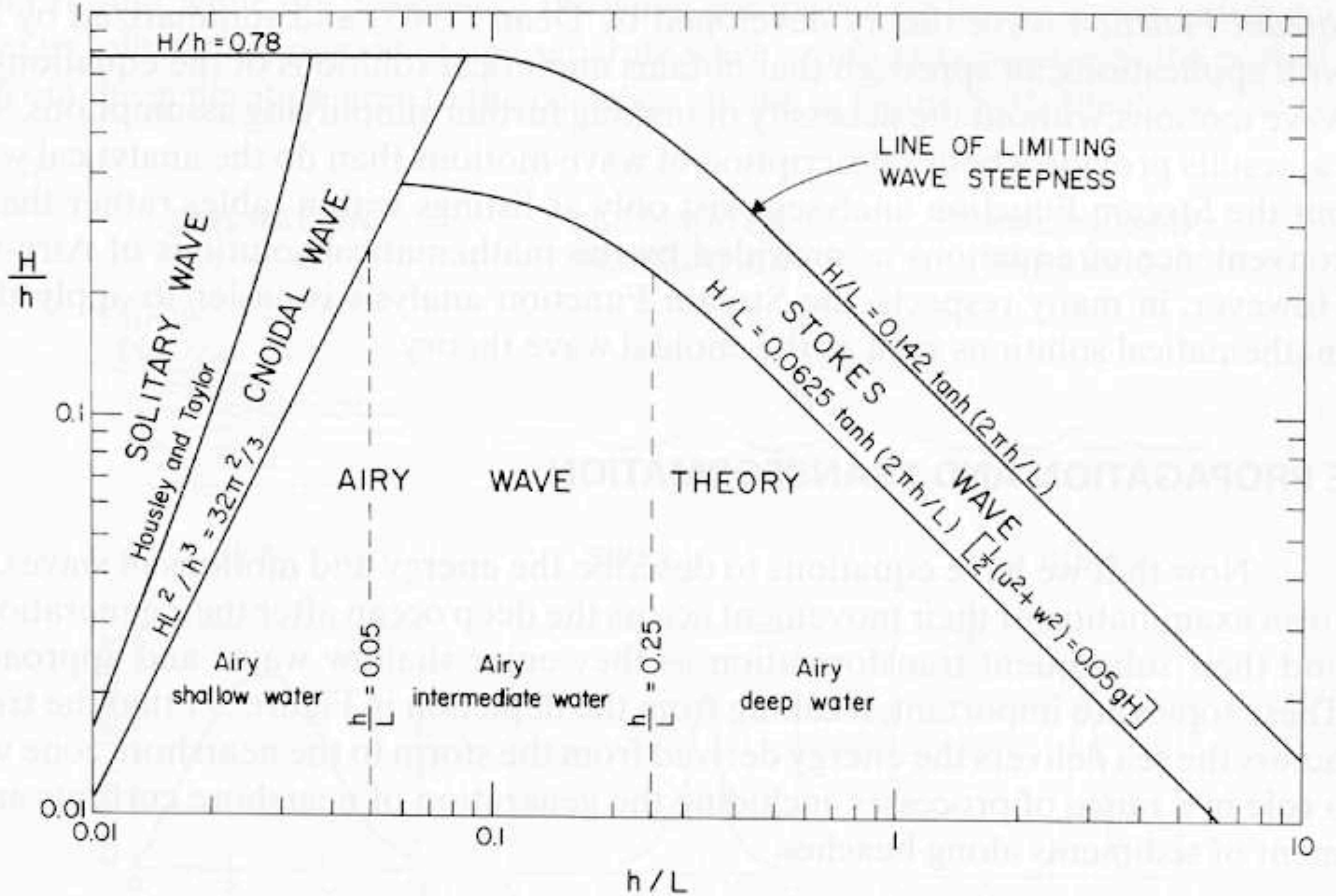
$$\left(\frac{H_0}{L_0}\right)_{max} = 0.142 = \frac{1}{7}$$

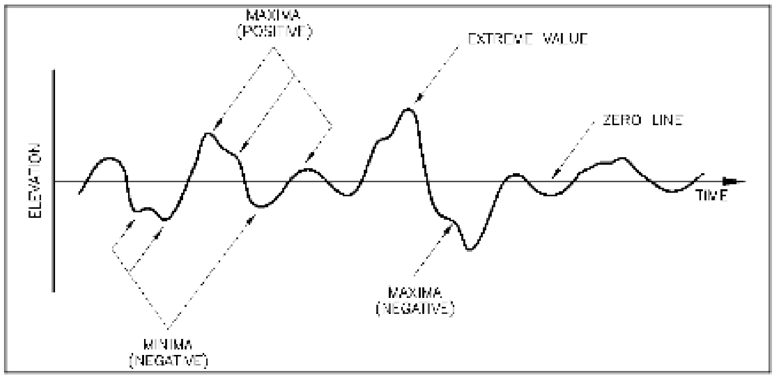
Stokes: finite amplitude wave theory

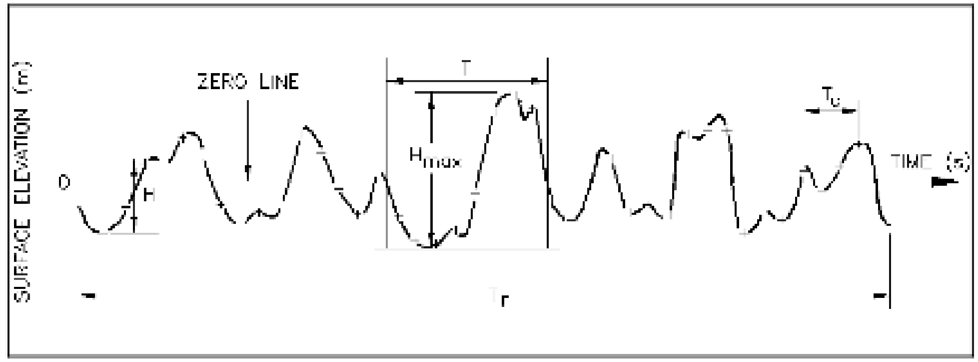
$$\gamma_b = \frac{H_b}{d_b}$$

Breaker Index ~= 0.78 :Solitary Wave Theory

Range of Wave Theories







Definition of Wave Parameters:

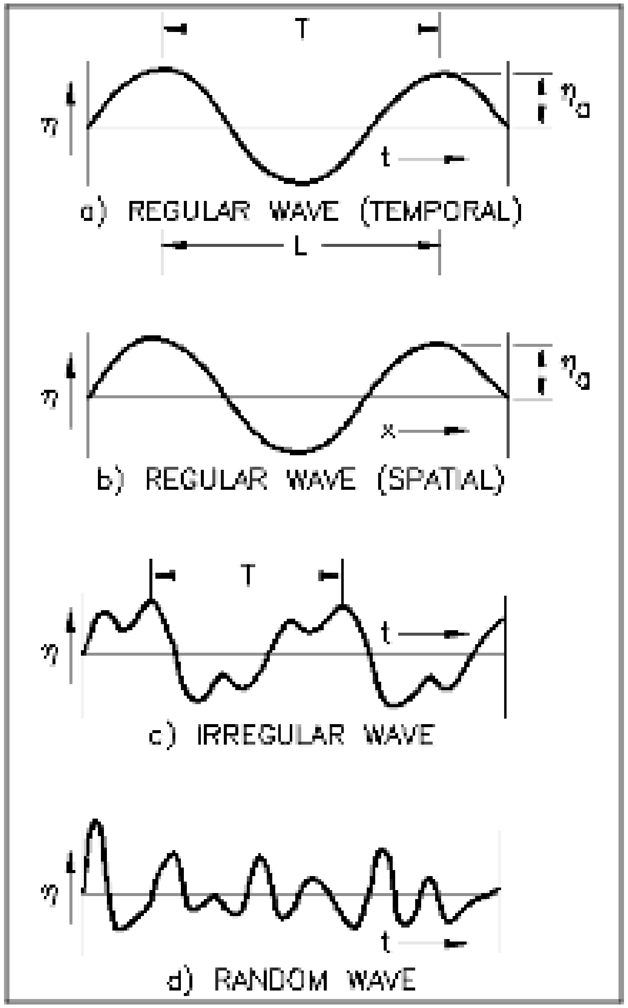
Hc, Tc: Characteristic wave height and period.

Hmax, Hmean: Maximum and mean wave heights

Hrms: root-mean-square height. $$H_{rms} = \sqrt{\frac{1}{N}\sum_{j=1}^{N} H_j^2}$$

Hs: significant wave height, average of the 1/3 largest waves in the record.

$$H_s = \frac{1}{\frac{N}{3}}\sum_{i=1}^{N/3} H_i$$

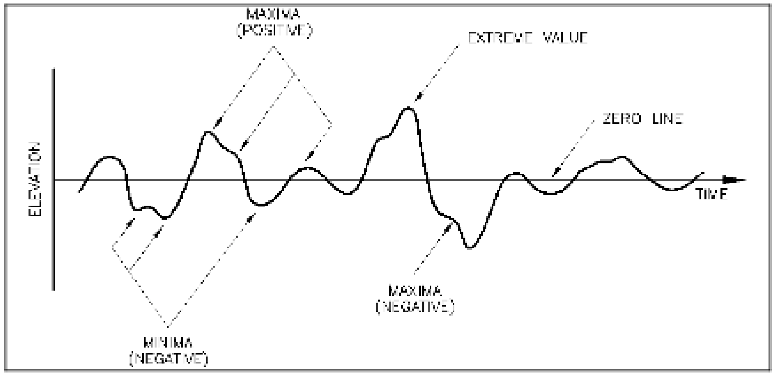


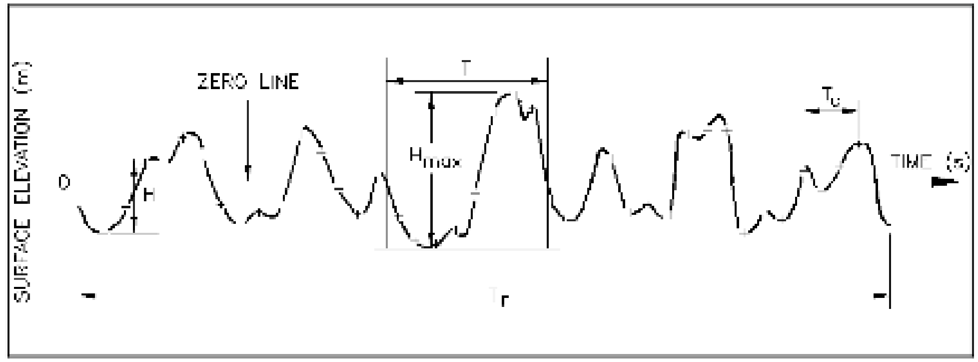
Irregular Waves

Methods of Analysis:

1) **Spectral Methods:** based on Fourier Transform of the sea surface

2) **Wave-by-wave (Wave Train) Analysis:** more simplified analysis of the time history of the sea state at a point



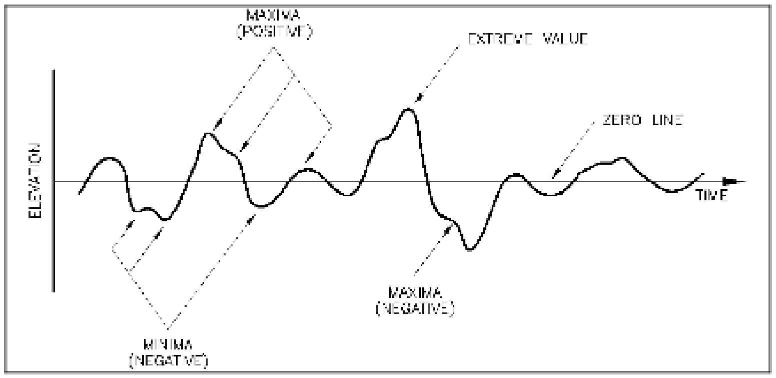


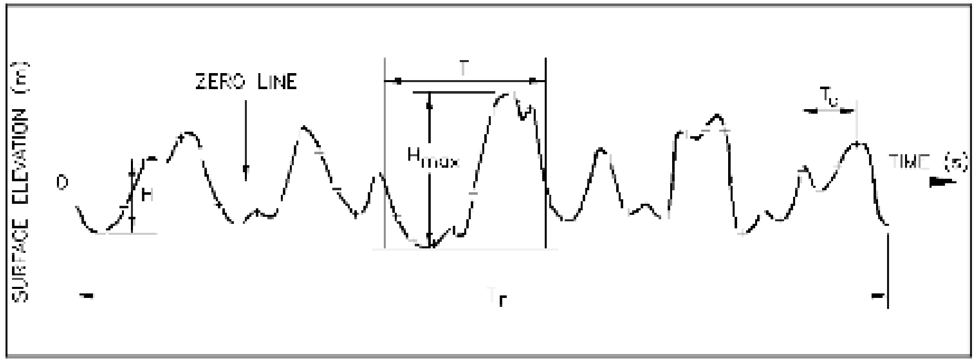
Wave-by-Wave (Wave Train):

Identify local maxima and minima.

Section the record into discrete waves.

Waves are determined by *zero up crossings or down crossings*.





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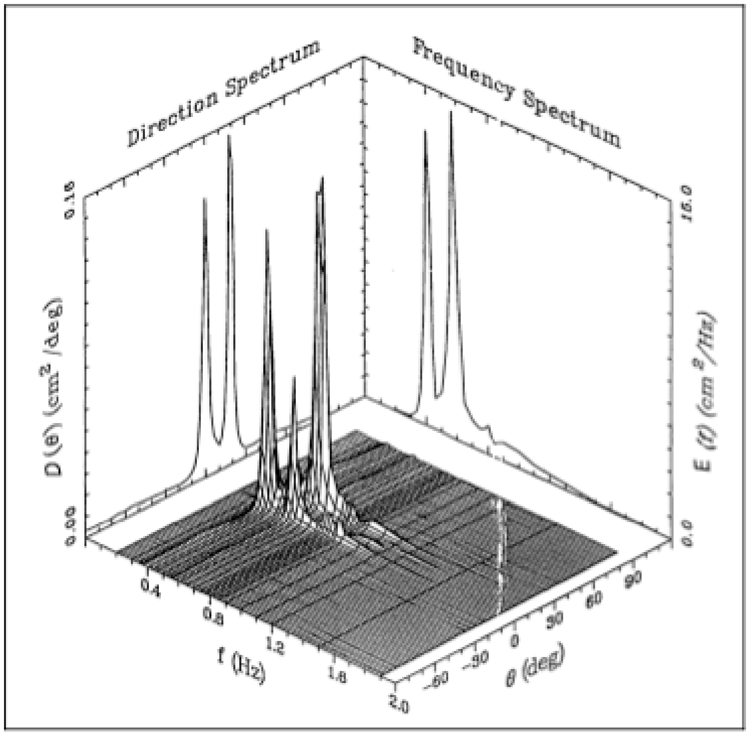
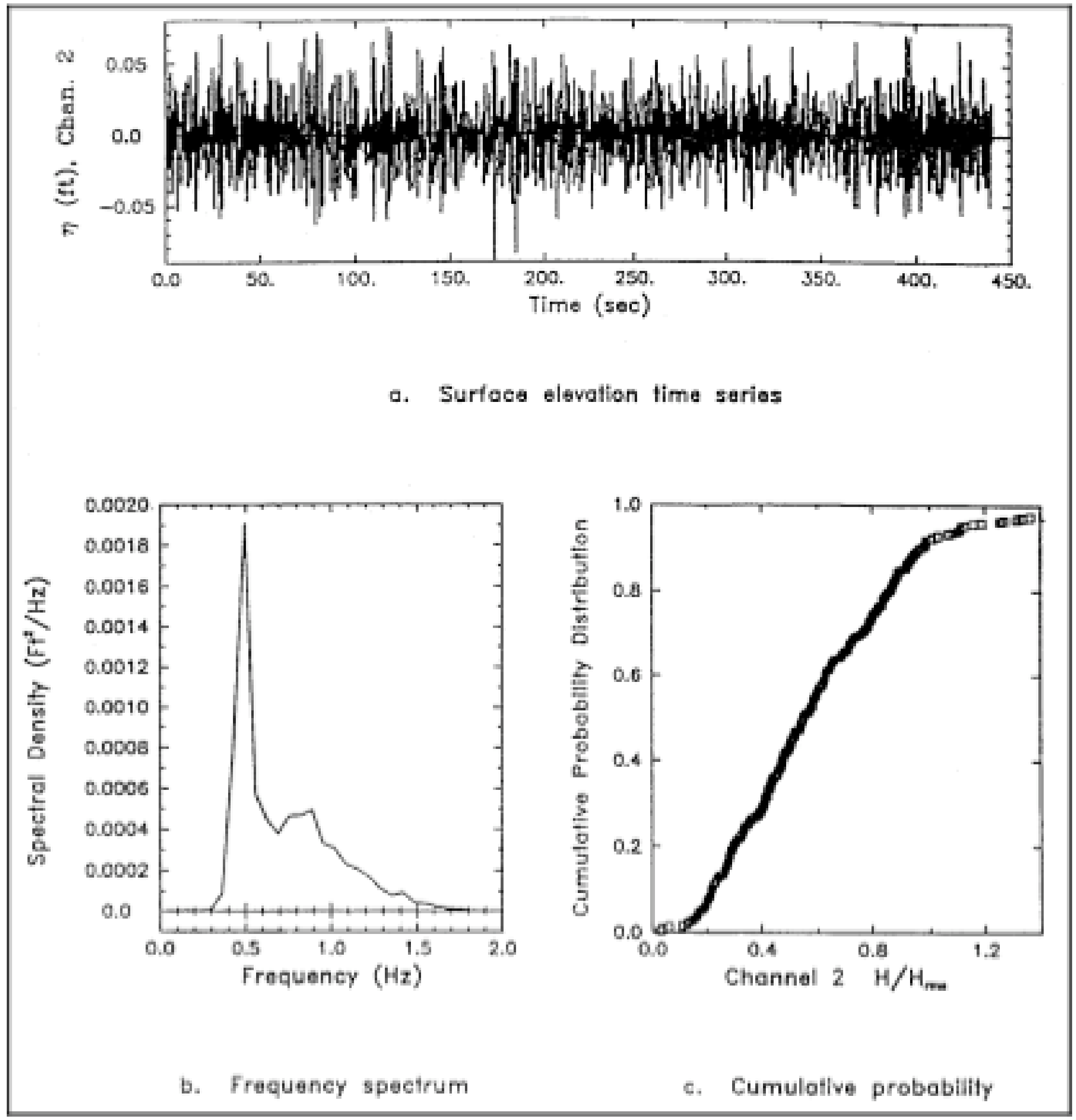
Hs: significant wave height, average of the 1/3 largest waves in the record.

$$H_s = \frac{1}{\frac{N}{3}} \sum_{i=1}^{N/3} H_i$$

Spectral Analysis:

Decomposition of sea surface into components, compute the distribution of wave energy and statistics for each frequency





a. Surface elevation time series

b. Frequency spectrum

c. Cumulative probability